Spring 2005 Math 182 Exam 2B

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If you cannot complete a problem (perhaps because you forgot a formula) but you think you know how, please describe. Correct methods will receive partial credits.

1. Determine whether each statement is TRUE or FALSE (2 points for correct answer, 1 for no answer, and 0 for wrong answer).

(a) The improper integral \( \int_1^\infty \frac{1}{x + e^{2x}} \, dx \) is

\[
\text{True} \quad \frac{1}{x + e^{2x}} < \frac{1}{e^{2x}} \quad (\text{this inequality is true}).
\]

(b) The \( x \)-coordinate of the center of mass, \( \overline{x} \), of the figure above is greater than 1.

False

(c) The pdf of the normal distribution with mean 0 and variance 1 is given by

\[
f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}.
\]

Then \( \int_{-\infty}^{\infty} f(x) \, dx = 1 \) (don’t try to integrate).

True. Definition of pdf

(d) The arc length of the curve \( y = x^3/3 + 1/x \) for \( 1 \leq x \leq 3 \) is given by

\[
L = \int_1^3 \sqrt{x^4 + (\ln x)^2} \, dx.
\]

False. \( x' = 1 \) and \( y' = x^2 - x^{-2} \). \( L = \int_1^3 \sqrt{1 + (x^2 - x^{-2})^2} \, dx \)

2. Show that the integral \( \int_0^1 \frac{1}{x} \, dx \) is divergent by showing that the appropriate limit does not exist.

We look at \( \lim_{t \to 0^+} \int_t^1 \frac{1}{x} \, dx \). Since the antiderivative of \( \frac{1}{x} \) is \( \ln x \),

\[
\lim_{t \to 0^+} \int_t^1 \frac{1}{x} \, dx = \lim_{t \to 0^+} (\ln 1 - \ln t).
\]

But \( \lim_{t \to 0^+} \ln t \) does not exist (or is \( -\infty \))

so the integral is divergent (i.e. does not have a finite value).
3. Set up an integral to find the area of the region bounded by \( y = x^2 + 1 \) and \( y = \frac{x^2}{2} + 1.5 \), shown below.

\[
A = \int_{-1}^{1} \left( \frac{x^2}{2} + 1.5 - (x^2 + 1) \right) \, dx
\]

\[
= 2 \int_{0}^{1} \left( \frac{x^2}{2} + 1.5 - (x^2 + 1) \right) \, dx
\]

4. Set up an integral to find the volume of the solid obtained by rotating the region bounded by \( y = \ln x \), \( y = 0 \), \( x = 1 \), and \( x = 2 \) about the \( x \)-axis.

(a) about the \( x \)-axis.

\[
\text{vol} = \int_{1}^{2} (\ln x)^2 \pi \, dx
\]

(b) about the \( y \)-axis. outer radius: 2. inner radius: \( e^y \).

\[
\text{vol} = \int_{0}^{2} \left[ 2\pi - (e^y)^2 \pi \right] \, dy
\]

5. The probability that a fair coin lands heads or tails should be 1/2. However, the probability \( X \) that a randomly selected coin (which may not be fair) will land heads can be modeled according to the probability density function \( (pdf) \)

\[
f(x) = \begin{cases} 
6x(1-x) & \text{for } 0 \leq x \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]

(a) Show that \( f(x) \) is a legitimate pdf. A sketch may help.

\[
\int_{0}^{1} 6x(1-x) \, dx = 1.
\]

(b) What is the probability that \( X \) is more than 2/3? Write an integral that represents the probability.

\[
P(X > \frac{2}{3}) = \int_{\frac{2}{3}}^{1} 6x(1-x) \, dx = 6 \left[ \frac{1}{2} x^2 - \frac{1}{3} x^3 \right]_{\frac{2}{3}}^{1} = 1 - \frac{20}{27} = \frac{7}{27}.
\]
6. A force of 21N is required to maintain a spring stretched from its natural length of 12cm to a length of 15cm. How much work is done in stretching the spring from 12cm to 20cm?

\[
\text{force} = k \cdot x, \quad \text{so} \quad 21N = k \cdot 3cm, \quad k = \frac{21 \text{ N}}{3 \text{ cm}} = 7 \text{ N/cm}.
\]

\[
\text{work} = \int_{12}^{20} 7 \cdot x \, dx = \frac{7}{2} x^2 \bigg|_{12}^{20} = 224 \text{ N} \cdot \text{cm} = 2.24 \text{ Nm}. \quad (J)
\]

7. The table (supplied by San Diego Gas and Electric) gives the power consumption in gigawatts in San Diego County from midnight to 6:00am on December 8, 1999. It is known that power is the derivative of energy.

<table>
<thead>
<tr>
<th>( t ) (hours after midnight)</th>
<th>( P ) (gigawatts)</th>
<th>( t ) (hours after midnight)</th>
<th>( P ) (gigawatts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.81</td>
<td>4.0</td>
<td>1.62</td>
</tr>
<tr>
<td>1.0</td>
<td>1.69</td>
<td>5.0</td>
<td>1.75</td>
</tr>
<tr>
<td>2.0</td>
<td>1.64</td>
<td>6.0</td>
<td>2.05</td>
</tr>
<tr>
<td>3.0</td>
<td>1.60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Use your choice of approximate integration to estimate the energy used during that time period.

\[
\text{Total energy} = \int_0^6 \text{power} \, dt.
\]

\[
\text{Total energy} \approx 1.81 + 1.69 + 1.64 + 1.60 + 1.62 + 1.75 = 10.1 \text{ gigawatts-hour}.
\]

(b) Using your answer from part (a), find the average power used between midnight and 6:00am.

\[
\text{avg. power} = \frac{1}{6} \int_0^6 \text{power} \, dt.
\]

\[
= \frac{10.1}{6} = 1.685
\]

If you need to know your updated grade by tomorrow (the last day to drop), please put your e-mail address here ________________________________.