Math 182 Quiz 10 (April 12)

Name: \\

If you cannot complete a problem (perhaps because you forgot a formula) but you think you know how, please describe. Correct methods will receive partial credits.

1. Find a power series representation for the function

\[
f(x) = \frac{1}{1 + 3x} = \frac{1}{1 - (-3x)} = \sum_{n=0}^{\infty} (-3x)^n.
\]

What is the interval of convergence?

we need \(|-3x| < 1\),
so \(|x| < \frac{1}{3}\).
\(-\frac{1}{3} < x < \frac{1}{3}\)

2. The graph of \(f\) is shown. Determine

(a) the sign of \(f(1.5)\). 
\(> 0\) about 1.2

(b) the sign of \(f'(1.5)\). 
\(< 0\)

(c) whether the series
\(1.2 + 0.8(x - 1.5) + 0.2(x - 1.5)^2 + \cdots\)
may be the Taylor series of \(f\) at \(x = 1.5\).

Taylor series at 1.5 should have negative linear and quadratic coefficients.

3. We know that the Taylor series of \(e^x\) at \(a = 0\) is

\[
e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots
\]

Using the above formula, write the Taylor series of \(e^{2x}\) at \(a = 0\).

\[
e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \cdots
\]