If you cannot complete a problem (perhaps because you forgot a formula) but you think you know how, please describe. Correct methods will receive partial credits.

1. Determine whether each statement is true or false and circle your answer (2 points for correct answer, 1 for “I don’t know”, and 0 for wrong answer).
   
   (a) The part of the plane \( z = 5 - x \) that lies inside the cylinder \( x^2 + y^2 = 1 \) can be parametrized as \( x = r \cos \theta, \ y = r \sin \theta, \) and \( z = 5 - r \cos \theta, \) where \( 0 \leq r \leq 1 \) and \( 0 \leq \theta \leq 2\pi. \)
   
   True  False  I don’t know

   (b) The domain of \( g(x, y) = \sqrt{1 - x^2 - y^2} \) is all \( (x, y) \) so that \( 1 \leq x^2 + y^2 \).
   
   True  False  I don’t know

   (c) If \( f(x, y) \to L \) as \( (x, y) \to (a, b) \) along every parabola through \((a, b), \) then 
   \( \lim_{(x,y)\to(a,b)} f(x, y) = L. \)
   
   True  False  I don’t know

2. Find the velocity and position vectors of a particle that has the given acceleration and the given initial velocity and position.

   \( a(t) = (0, 0, -8), \ v(0) = (1, 2, 0), \ r(t) = (-2, 3, 1). \)

3. Show that the following limit does not exist.

   \[ \lim_{(x,y)\to(0,0)} \frac{4xy}{2x^2 + y^2}. \]
4. Use the table of values of $f(x, y)$ to answer the following.

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(a) Estimate $f_x(2, 2)$.

(b) Is $f_{xy}(2, 2)$ positive or negative? (extra 2 points: what is its approximate value? (justification required for extra credit))

(c) Estimate the value of $D_uf(2, 2)$, where $u = (i + j)/\sqrt{2}$.

5. If $R$ is the resistance of two resistors, connected in parallel, with resistances $R_1$ and $R_2$, then it is known that

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2},$$

which is equivalent to $R = \frac{R_1R_2}{R_1 + R_2}$.

If the resistances are measured in ohms as $R_1 = 24\Omega$, and $R_2 = 40\Omega$ with a possible error of 0.8% in each case, estimate the maximum error in the calculated value of $R$ following the steps.

(a) Find the largest error in each of $R_1$ and $R_2$.

(b) Find the differential $dR$ (use the second formula above; note the symmetry in $R_1$ and $R_2$).

(c) Evaluate the differential in part (b) so that you obtain the maximum error in $R$ measurement. If you could not do part (b), use (incorrect) $dR = \frac{R_2}{R_1 + R_2}dR_1 + \frac{R_1}{R_1 + R_2}dR_2$. 

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6. Answer the following questions regarding a surface \( z = x \ln y \) near \((2, e, 2)\).

(a) Find an equation of the tangent plane to the surface \( f(x, y) = x \ln y \) at \((2, e)\).

(b) Use the equation from part (a) to approximate the function value \( f(2.03, 3) \).

(c) Find the directional derivative of \( f(x, y) = x \ln y \) at \((2, e)\) in the direction of \( \mathbf{u} = \left\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle \).