1. Determine whether each statement is true or false and circle your answer (2 points for correct answer, 1 for "I don't know", and 0 for wrong answer). Extra credit of 1 point each for correcting false statements/giving counterexamples.

(a) Suppose \((1,3)\) is a critical point of a function \(f\) with continuous second derivatives. If \(f_{xx} = 1\), \(f_{yy} = 6\) and \(f_{xy} = 3\), then \(f\) has a local minimum at \((1,3)\).

\[ D = 1 \cdot 6 - 3^2 = -3 < 0. \] saddle pt.

(b) The function \(g(x, y) = x - 3y + 5\) has an absolute maximum value in \([0,1] \times [0,1]\).

True False I don't know

Any nonconstant continuous function has a max & min in a closed & bounded domain.

(c) When estimating \(\iint_R f(x, y)\,dA\) on \(R = [0,2] \times [0,4]\) using the Midpoint Rule with \(m = n = 2\), one of the needed function values is \(f(0,1)\).

True False I don't know

values needed are \(f(0.5, 1), f(1.5, 3), f(1.5, 1), f(1.5, 3)\).

(d) The two integrals are equal:

\[ \int_0^1 \int_0^1 e^{xy} \,dx\,dy \quad \text{and} \quad \int_0^1 \int_y^1 e^{xy} \,dx\,dy \]

True False I don't know

\[ \text{Area of integration is not linear.} \]

Right integral is equal to
\[ \int_0^1 \int_0^y e^{xy} \,dx\,dy \]
2. Write (but do not evaluate) \( \iint_D \sin(x^2 + y^2) \, dA \) in polar coordinates if \( D \) is the region that lies to the left of the \( y \)-axis between the circles \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = 9 \).

\[
\sin(x^2 + y^2) = \sin(r^2), \quad dA = r \, dr \, d\theta.
\]

\[
\int_{\pi}^{3\pi/2} \int_{1}^{3} \sin(r^2) \cdot r \, dr \, d\theta
\]

3. Answer questions regarding the following integral.

\[
\int_{1}^{2} \int_{0}^{2} (2x + y) \, dx \, dy
\]

(a) Sketch the region over which the integral is taken.

(b) Evaluate the integral.

\[
\left. \int_{1}^{2} \int_{0}^{2} (2x + y) \, dx \, dy = \int_{1}^{2} (x^2 + xy) \bigg|_{y}^{2} \, dy \right.
\]
\[
= \int_{1}^{2} [(4z + 2y) - (y^2 + y^2)] \, dy = \int_{1}^{2} (4z + 2y - 2y^2) \, dy
\]
\[
= 4y + y^2 - \frac{2}{3} y^3 \bigg|_{1}^{2} = 8 + 4 - \frac{16}{3} - (4 + 1 - \frac{2}{3})
\]
\[
= -\frac{14}{3} = -\frac{7}{3}
\]
4. The contours of \( f(x, y) = x^2y \) and the ellipse \( x^2 + 2y^2 = 6 \) are shown below. The ellipse is used only in part (c).

(a) What are critical points of \( f(x, y) \)? From the level curves shown, what can you say about the nature of the critical points?

\[
\begin{align*}
\frac{\partial f}{\partial x} &= 2xy = 0 \quad \text{for } x=0 \text{ or } y=0, \\
\frac{\partial f}{\partial y} &= x^2 = 0 \quad \Rightarrow x=0
\end{align*}
\]

\((0,0)\), saddle point.

(b) Based on the level curves shown, at what point(s) in \([-4,4] \times [-4,4]\) does \( f \) attain the absolute maximum value? the absolute minimum value?

Abs. Max \( a+ (-4,4) \) and \((4,4)\) & abs. Min \( a+ (-4,-4) \) and \((4,-4)\)

(c) Using the figure as an aid, find the maximum and minimum values of \( f(x,y) = x^2y \) under the constraint \( x^2 + 2y^2 = 6 \).

\[
\begin{align*}
\frac{\partial f}{\partial x} &= 2xy = 0 \quad \Rightarrow 2x = 0 \quad \text{for } x=0 \quad \text{or } y=0, \\
\frac{\partial f}{\partial y} &= x^2 = 0 \quad \Rightarrow x = 0, \quad \text{from } (1) y^2=3 \quad \text{so } y = \pm \sqrt{3}. \\
\text{Constraint} \quad x^2 + 2y^2 = 6 \quad \Rightarrow x = \pm 2.
\end{align*}
\]

<table>
<thead>
<tr>
<th>Points</th>
<th>( f ) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, \sqrt{3}))</td>
<td>0</td>
</tr>
<tr>
<td>((0, -\sqrt{3}))</td>
<td>0</td>
</tr>
<tr>
<td>((\pm 2, +1))</td>
<td>4, max</td>
</tr>
<tr>
<td>((\pm 2, -1))</td>
<td>-4, min</td>
</tr>
</tbody>
</table>

\[ \sqrt{2} \]
5. Set up the integral to find the surface area of parametrized surface \( \mathbf{r}(u,v) = (v^2, uv, u^2/2) \), where \( 0 \leq u \leq 2 \) and \( 0 \leq v \leq 1 \), following the steps.

(a) Find the two partial derivatives \( \mathbf{r}_u \) and \( \mathbf{r}_v \).

\[
\mathbf{r}_u = \langle 0, v, u \rangle \\
\mathbf{r}_v = \langle 2v, u, 0 \rangle
\]

(b) Find \( \mathbf{r}_u \times \mathbf{r}_v \), and \( |\mathbf{r}_u \times \mathbf{r}_v| \).

\[
\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & v & u \\ 2v & u & 0 \end{vmatrix} = -u^2 \mathbf{i} - (-2uv) \mathbf{j} - 2v^2 \mathbf{k}.
\]

\[
|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{u^4 + 4u^2v^2 + 4v^4} = \sqrt{(u^2 + 2v^2)^2} = u^2 + 2v^2.
\]

(c) Set up the integral to calculate the surface area. Do not evaluate.

\[
\iint_{D} \sqrt{u^4 + 4u^2v^2 + 4v^4} \, du \, dv
\]

6. Set up the double integral that represents the volume of the solid enclosed by the paraboloid \( z = x^2 + 3y^2 \) and the planes \( x = 0 \), \( y = 1 \), \( y = x \), and \( z = 0 \). Do not evaluate.

\[
\int_{0}^{1} \int_{0}^{y} (x^2 + 3y^2) \, dx \, dy
\]

\[
\text{or} \quad \int_{0}^{1} \int_{0}^{x} (x^2 + 3y^2) \, dy \, dx
\]