1. Rewrite (but do not evaluate) \( \iiint_{H} \sin(x^2 + y^2 + z^2) \, dV \) using spherical coordinates if \( H \) is the hemispherical region that lies above the \( xy \)-plane and below the sphere \( x^2 + y^2 + z^2 = 4 \).

\[
\int_{0}^{\pi/2} \int_{0}^{\pi} \int_{0}^{2} \sin(r^2) r^2 \sin \phi \, dr \, d\phi \, d\theta.
\]

2. Consider a two dimensional vector field given by \( \mathbf{F}(x, y) = (y + 2x)i + xj \).

(a) Show that \( \mathbf{F}(x, y) \) above is a conservative vector field.

\[
\frac{\partial Q}{\partial y} = 1, \quad \frac{\partial P}{\partial x} = 1.
\]

(b) Find a scalar valued function \( f(x, y) \) for which \( \nabla f(x, y) = \mathbf{F}(x, y) \) above.

\[
\nabla f = y + 2x \Rightarrow f = \int (y + 2x) \, dx = x^2 + xy + g(y)
\]

\[
f_y = x \quad \text{(check)} \quad f_y = x + g'(y) = x . \quad \text{so } g'(y) = 0
\]

\[
f(x, y) = x^2 + xy + C \quad \text{optional}
\]

(c) Using \( f(x, y) \) found in part (b), set up the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) of \( \mathbf{F}(x, y) \) along \( C \) parametrized by \( \mathbf{r}(t) = \sqrt{t} \mathbf{i} + (1 + t^3) \mathbf{j} \), \( 0 \leq t \leq 1 \) using the fundamental theorem of line integrals. (If you could not do part (b), use \( f(x, y) = x^2 + y^2 \), which is wrong.)

\[
\mathbf{r}'(t) = \sqrt{t} \mathbf{i} + 3t^2 \mathbf{j}, \quad \mathbf{r}'(0) = 0 \mathbf{i} + 1 \mathbf{j}
\]

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \int_{0}^{1} f(1, t) - f(0, 1) \, dt = 1^2 + 1 \cdot 2 + C - (0^2 + 0.1 + C)
\]

\[
= 3 + C - 0 - C = 3.
\]

(C. cancels anyway.)
3. Consider the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) of a vector field \( \mathbf{F}(x, y) = yi + xyj \) along some curve \( C \) parametrized by \( \mathbf{r}(t) \).

(a) If \( C_1 \) is parametrized by \( \mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} \) for \(-1 \leq t \leq 1\), set up (but do not integrate) the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) along \( C_1 \). Please work on your integrand until you have a scalar function.

\[
\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{-1}^{1} \langle t^2, t^3 \rangle \cdot \langle 1, 2t \rangle \, dt
\]

\[
= \int_{-1}^{1} (t^2 + 3t^4) \, dt
\]

(b) If \( C_2 \) is parametrized by \( \mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} \) for \( 0 \leq t \leq 2\pi \), set up (but do not evaluate) the line integral \( \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \) using Green's Theorem.

\[
\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA
\]

\[
= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (y - 1) \, dy \, dx
\]

since \( \frac{\partial Q}{\partial x} = y \), \( \frac{\partial P}{\partial y} = 1 \)
4. Determine whether each statement is true or false and circle your answer (2 points for correct answer, 1 for "I don't know", and 0 for wrong answer).

(a) In spherical coordinates, \( dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \).

   True ∟ False ∟ I don't know

(b) If \( \mathbf{F}(x, y) \) is a vector field, then \( \text{div} \, \mathbf{F} \) is a vector field.

   True ∟ False ∟ I don't know

(c) The vector field given by \( \mathbf{F}(x, y) = (1 + \ln y)i + x^2 j \) is conservative vector field.

   True ∟ False ∟ I don't know

(d) The divergence of the vector field \( \mathbf{i} + (x + yz)j + (zx - y)k \) is \( 3x + y \).

   True ∟ False ∟ I don't know

5. Set up (but do not evaluate) a triple integral to find the volume of the solid under the surface \( z = x^2y \) and above the triangle in the \( xy \)-plane with vertices (0,0), (0,2) and (4,0).

\[
\text{Vol} = \iiint_E 1 \, dV \\
= \int \int \int_{E} 1 \, dV \\
= \int_{0}^{2} \int_{0}^{4} \int_{0}^{x^2y} 1 \, dz \, dy \, dx \\
= \int_{0}^{2} \int_{0}^{4} x^2y \, dx \, dy \\
= \int_{0}^{2} \int_{0}^{4} x^2y \, dx \, dy \\
= \text{(Volume calculation)}
\]
6. Set up (but do not evaluate) the surface integral \[ \iint_S \mathbf{F} \cdot d\mathbf{S} \] where \( \mathbf{F}(x, y, z) = xi + xyj + zk \) and \( S \) is the part of the plane \( 2x + y + z = 2 \) that lies in the first octant, oriented upward.

Use \[ \iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div} \mathbf{F} \, dV \] for best results.

\[ \begin{aligned} \iint_S \mathbf{F} \cdot d\mathbf{S} &= \iint_D \left( \frac{\partial z}{\partial x} - Q \frac{\partial z}{\partial y} + R \right) \, dA \\ &= \int_0^2 \int_0^{2-2x} \left( -x(-z) - xy(-1) + 2z-2x-y \right) \, dy \, dx \\ &= \int_0^2 \left( x^2 - 2x \right) \, dx \end{aligned} \]

7. If the surface \( S \) in Problem 6 had been the boundary surface of the solid \( E \) bounded by the planes \( x = 0, y = 0, z = 0 \) and \( 2x + y + z = 2 \) (so that \( S \) is closed), then the surface integral \( \iint_S \mathbf{F} \cdot d\mathbf{S} \) equals \( \iiint_E \text{div} \mathbf{F} \, dV \) by the divergence theorem, with \( \mathbf{F}(x, y, z) = xi + xyj + zk \) as in problem 6. Set up (but do not evaluate) the triple integral here.

\[ \iiint_E \text{div} \mathbf{F} \, dV = \int_0^2 \int_0^{2-2x} \int_0^{2-z} (x+y+z) \, dz \, dy \, dx \]

8. (extra credit: 5pts) Shown below is a sketch of the conservative vector field \( \mathbf{F} = \nabla f \) for some \( f \). Add to the figure at least 3 level curves of \( f(x, y) \). (Hint: what's the relationship between gradient \& level curves since at any point \((x_0, y_0)\), \( \nabla f \) points in the direction of greatest increase of \( f \) values.

level curves of \( f \) and the gradient vectors of \( f \)?)