Solutions

6. (a) \( f_x(-1,2) < 0 \) since the surface goes down as we move toward positive \( x \) direction from \((-1,2)\).

(b) \( f_y(-1,2) < 0 \)

(c) \( f_{xx}(-1,2) \) is the concavity along \( x = -1 \) near \((-1,2)\). So \( f_{xx}(-1,2) > 0 \)

(d) \( f_{yy}(-1,2) \) is concavity along \( y = 2 \) near \((-1,2)\).
\[ f_{yy}(-1,2) < 0 \]

\[ \text{8. We see that } f(2,1) = 10, \ f(2.5,1) = 12, \ f(1.2,1) = 8 \]
\[ f_x(2,1) \text{ can be approximated by any of} \]
\[ \frac{f(2.1) - f(1.2,1)}{2-1.2} = \frac{10-8}{0.8} = 2.5, \]
\[ \frac{f(2.5) - f(2,1)}{2.5-2} = \frac{12-10}{0.5} = 4, \quad \text{or} \]
\[ \frac{f(2.5,1) - f(1.2,1)}{2.5-1.2} = \frac{12-8}{1.3} \approx 3.1 \text{ closest to the true value} \]

\[ f_x(2,1) \text{ is approximated by} \]
\[ \frac{f(2,1) - f(2,0)}{1-0} = \frac{10-12}{1} = -2 \]
\[ \frac{f(2,1.9) - f(2,1)}{1.9-1.1} = \frac{8-10}{0.9} = -2.2 \]
\[ \frac{f(2,1.9) - f(2,0)}{1.9-0} = \frac{8-12}{1.9} = -2.1 \]

Note \( f_y(2,1) \) approximations are closer in values than those for \( f_x(2,1) \). This is because the function changes more steeply in \( x \) direction.
11.3 9  \[ f(x, y) = 16 - 4x^2 - y^2. \]

\[ f_x(x, y) = -8x \quad \text{so} \quad f_x(1, 2) = -8. \]
\[ f_y(x, y) = -2y \quad \text{so} \quad f_y(1, 2) = -4. \]

(See Figures 2 & 3 on p. 760)

14  \[ f(x, y) = x^5 + 3x^3y^2 + 3xy^4 \]

\[ f_x(x, y) = 5x^4 + 9x^2y^2 + 3y^4. \]
\[ f_y(x, y) = 6x^3y + 12xy^3 \]

24  \[ f(x, y) = \int_y^x \cos(t^2) \, dt \quad \text{requires the Fundamental Theorem of Calculus}. \]

\[ \frac{\partial f}{\partial x}(x, y) = \cos(x^2). \]

\[ \frac{\partial f}{\partial y}(x, y) = -\cos(y^2) \quad \text{(minus because y is above, lower limit)} \]

36  \[ f(x, y) = \sin(2x + 3y). \]

\[ f_y(x, y) = 3\cos(2x + 3y). \]
\[ f_y(-6, 4) = 3\cos(-12 + 12) = 3\cos0 = 3. \]