11.5 2. \( z = x \ln(x + 2y) \), \( x = \sin t \), \( y = \cos t \)

\[
\frac{dz}{dt} = \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}
\]

where

\[
\frac{\partial z}{\partial x} = \ln(x + 2y) + \frac{x}{x + 2y}
\]

by product rule,

\[
\frac{\partial z}{\partial y} = \frac{2x}{x + 2y} \cdot z = \frac{2x}{x + 2y}
\]

\[
\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = -\sin t.
\]

\[
\frac{dz}{dt} = \left( \ln(x + 2y) + \frac{x}{x + 2y} \right) \cdot \cos t - \frac{2x}{x + 2y} \sin t.
\]

3. \( z = \sin \alpha \tan \beta \), \( \alpha = 3s + t \), \( \beta = 4t - s \)

\[
\frac{\partial z}{\partial \alpha} = \cos \alpha \tan \beta, \quad \frac{\partial z}{\partial \beta} = \sin \alpha \sec^2 \beta
\]

\[
\frac{\partial \alpha}{\partial s} = 3, \quad \frac{\partial \beta}{\partial s} = 1.
\]

This makes

\[
\frac{\partial z}{\partial s} = \frac{\partial z}{\partial \alpha} \frac{\partial \alpha}{\partial s} + \frac{\partial z}{\partial \beta} \frac{\partial \beta}{\partial s} = 3 \cos \alpha \tan \beta + \sin \alpha \sec^2 \beta.
\]

\[
\frac{\partial z}{\partial t} = 1 \quad \text{and} \quad \frac{\partial \beta}{\partial t} = -1 \quad \text{so}
\]

\[
\frac{\partial z}{\partial t} = \frac{\partial z}{\partial \alpha} \frac{\partial \alpha}{\partial t} + \frac{\partial z}{\partial \beta} \frac{\partial \beta}{\partial t} = \cos \alpha \tan \beta - \sin \alpha \sec^2 \beta.
\]

17. \( z = x^2 + xy^3 \), \( x = uv^2 + u^3 \), \( y = u + v + w \)

\[
\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial u}, \quad \text{and} \quad x = 2, \ y = 3 \ \text{when} \ u = 2, \ v = 1, \ w = 0
\]

\[
\frac{\partial z}{\partial x} = 2x + 3, \quad \text{so} \quad \frac{\partial z}{\partial x} = 31 \ \text{when} \ u = 2, \ v = 1, \ w = 0
\]

\[
\frac{\partial z}{\partial y} = 3xy^2, \quad \text{so} \quad \frac{\partial z}{\partial y} = 54
\]

\[
\frac{\partial x}{\partial u} = v^2, \quad \text{so} \quad \frac{\partial x}{\partial u} = 1
\]

\[
\frac{\partial y}{\partial u} = 1 + (x + z)te^{x^2} \quad \text{when} \ u = 2, \ v = 1, \ w = 0
\]

\[
\frac{\partial z}{\partial u} = 85
\]

To be continued on the next page.
\[
\frac{\partial^2 z}{\partial v^2} = \frac{\partial^2 z}{\partial x^2} \frac{\partial x}{\partial v} + \frac{\partial^2 z}{\partial y^2} \frac{\partial y}{\partial v}
\]
\[
\frac{\partial x}{\partial v} = 2uv \quad \text{so} \quad \frac{\partial x}{\partial v} = 4 \quad \text{when} \quad u=2, \quad v=1, \quad w=0
\]
\[
\frac{\partial y}{\partial v} = e^w \quad \text{so} \quad \frac{\partial y}{\partial v} = 1 \]
\[
\frac{\partial^2 z}{\partial v^2} = 31 \cdot 4 + 54 \cdot 1 = 178
\]
\[
\frac{\partial^2 z}{\partial w^2} = \frac{\partial^2 z}{\partial x^2} \frac{\partial x}{\partial w} + \frac{\partial^2 z}{\partial y^2} \frac{\partial y}{\partial w}
\]
\[
\frac{\partial x}{\partial w} = 3w^2 \quad \text{so} \quad \frac{\partial x}{\partial w} = 0 \quad \text{when} \quad u=2, \quad v=1, \quad w=0
\]
\[
\frac{\partial y}{\partial w} = v e^w \quad \frac{\partial y}{\partial w} = 1
\]
\[
\frac{\partial^2 z}{\partial w^2} = 31 \cdot 0 + 54 \cdot 1 = 54
\]

\[M = xe^{y-z^2}, \quad x=2uv, \quad y=u-v, \quad z=u+v, \quad M\]
\[
\frac{\partial M}{\partial u} = \frac{\partial M}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial M}{\partial y} \frac{\partial y}{\partial u} + \frac{\partial M}{\partial z} \frac{\partial z}{\partial u}, \quad \text{and}
\]
\[
x = -6 \quad \text{and} \quad y = 4 \quad \text{and} \quad z = 2 \quad \text{when} \quad u=3 \quad \text{and} \quad v=-1.
\]
\[
\frac{\partial M}{\partial x} = e^{y-z^2} \quad \text{so} \quad \frac{\partial M}{\partial x} = 1 \quad \text{when} \quad u=3 \quad \text{and} \quad v=-1.
\]
\[
\frac{\partial M}{\partial y} = xe^{y-z^2} \quad \text{so} \quad \frac{\partial M}{\partial y} = -6 \cdot 1 = -6
\]
\[
\frac{\partial M}{\partial z} = x \cdot (-2z)e^{y-z^2} \quad \text{so} \quad \frac{\partial M}{\partial z} = 24
\]
\[
\frac{\partial^2 M}{\partial x^2} = 2v \quad \frac{\partial^2 M}{\partial y^2} = -2
\]
\[
\frac{\partial^2 M}{\partial z^2} = 1
\]
\[
\frac{\partial^2 M}{\partial u^2} = 1 \cdot (-2) + (-6) \cdot 1 + 24 \cdot 1 = 16
\]

To be continued on the next page.
(20) continued. \[ \frac{\partial M}{\partial v} = \frac{\partial M}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial M}{\partial y} \frac{\partial y}{\partial v} + \frac{\partial M}{\partial t} \frac{\partial t}{\partial v}, \] and
\[
\frac{\partial x}{\partial v} = 2u, \quad \frac{\partial y}{\partial v} = -1, \quad \frac{\partial t}{\partial v} = 1.
\]

so \[ \frac{\partial M}{\partial v} = 1 \cdot 6 + (-6)(-1) + 24 \cdot 1 = 36 \]

(23) \[ \sqrt{xy} = 1 + x^2y. \] equation 6 says for \( F(x, y) = 0, \)

\[ \frac{dy}{dx} = -\frac{F_x}{F_y}. \]

\[ F(x, y) = \sqrt{xy} - 1 - x^2y = 0. \]

and \( F_x = \frac{1}{2}y(xy)^{-\frac{1}{2}} - 2xy, \)
\[ F_y = \frac{1}{2}x(xy)^{-\frac{1}{2}} - x^2. \]

so \[ \frac{dy}{dx} = -\frac{\frac{1}{2}y(xy)^{-\frac{1}{2}} - 2xy}{\frac{1}{2}x(xy)^{-\frac{1}{2}} - x^2}. \]

You're welcome to simplify this but it won't look very pretty anyway.

(25) \[ x^2 + y^2 + z^2 = 3xyzt. \] equation 7 says for \( F(x, y, z) = 0, \)

\[ \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \quad \text{and} \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}. \]

We can use \( F(x, y, z) = x^2 + y^2 + z^2 - 3xyzt = 0. \)

\[ F_x = 2x - 3yt, \]
\[ F_y = 2y - 3xt, \]
\[ F_z = 2z - 3xy. \]

\[ \frac{\partial z}{\partial x} = -\frac{2x - 3yt}{2z - 3xy}. \quad \text{or} \quad \frac{3yt - 2x}{2z - 3xy}. \]

\[ \frac{\partial z}{\partial y} = -\frac{2y - 3xt}{2z - 3xy} = \frac{3xt - 2y}{2z - 3xy}. \]