8. \( f(x, y) = \sqrt{x^2 + y^2} - 1 + \ln(4 - x^2 - y^2). \)

We need \( x^2 + y^2 - 1 \geq 0 \), \( \Rightarrow x^2 + y^2 \geq 1 \).

and \( 4 - x^2 - y^2 > 0 \), \( \Rightarrow x^2 + y^2 < 4 \),

not due to \( \ln 0 \) is not defined.

Our domain \( D \) is in both sets.

\( D \) is the domain area bounded by \( x^2 + y^2 \leq 1 \) and \( x^2 + y^2 = 4 \), but excluding the latter circle.

15. (a) \( f(x, y) = |x| + |y| \) has traces \( m x = k \) and \( y = k \) that are absolute value functions \( z = \text{const.} + |y| \) and \( z = \text{const.} + |x| \) respectively.

So, the graph of \( f(x, y) \) looks like this along the \( x \)-axis: so V.

(b) \( f(x, y) = |x| y \) has traces \( m x = k \) that are \( z = |k|y \).

The graph of \( f(x, y) \) is triangular along \( x \)-axis, touching the axis for every \( k \), so V.

(c) \( f(x, y) = \frac{1}{1 + x^2 + y^2} \).

Since \( x^2 \) and \( y^2 \) is in the denominator, as \( x \) gets large, the value of \( f \) gets smaller and similarly for \( y \). That is, \( f \) has maximum value at the origin. so I.
(d) \( f(x, y) = (x^2 - y^2)^2 \)  
Because of the square on the outside, \( f(x, y) = (x^2 - y^2)^2 = (y^2 - x^2)^2 \), i.e. \( f \) is symmetric in \( x \) and \( y \), so not \( II \) and \( III \) is sinusoidal (sine or cosine) so \( IV \).

(e) \( f(x, y) = (x - y)^2 \). Has traces in \( z = 0 \) of \( (x - y)^2 = 0 \) 
\[ \Rightarrow x = y. \] That is, the graph of \( f \) on the xy-plane is \( y = x \). So \( II \).

(f) \( f(x, y) = \sin(x|1 + |y|). \) The only graph that goes up and down enough to be that of sine is \( III \).

16 \( f(x, y) = \sqrt{16 - x^2 - 16y^2} \). Need \( 16 - x^2 - 16y^2 \geq 0 \) 
\[ \frac{x^2}{16} + y^2 \leq 1. \]  
\[ \Rightarrow \text{Domain is inside the ellipse } \frac{x^2}{16} + y^2 \leq 1. \]  
Passing thru \((4,0), (0,1), (4,0), (0,-1)\).

Traces in \( x=k \). 
\[ z = \sqrt{16 - k^2 - 16y^2} \]  
For \( k > 0 \), this is upper half of ellipse.

Traces in \( y=k \). 
\[ z = \sqrt{16 - x^2 - k^2} \]  
For \( k = 0 \), upper semi-circle of radius \( x \). For \( k \) between \( 0 \& 4 \), we have smaller circles.

Traces in \( z=k \). 
\[ k = \sqrt{16 - x^2 - 16y^2} \]  
For \( k = 0 \), we have the upper half of an ellipse. For \( k \) between \( 0 \& 4 \), we have smaller ellipses.

Quick and easy! Take \( z = \sqrt{16 - x^2 - 16y^2} \) & square both sides.
\[ z^2 = 16 - x^2 - 16y^2 \]  
so \( x^2 + 16y^2 + z^2 = 16 \) 
\[ \Rightarrow \frac{x^2}{16} + \frac{y^2}{1} + \frac{z^2}{16} = 1 \]  
so graph of \( f \) is the upper half of an ellipsoid.
If \( f(x, y) = x^2 - y^2 \), (see example 7.)

Traces in \( x = k \) are \( z = k^2 - y^2 \).

Traces in \( y = k \) are \( z = x^2 - k^2 \).

Traces in \( z = k \) are \( k = x^2 - y^2 \).

So the graph looks like

Traces in \( x = k \), are
(a) \( x^2 - y^2 = k^2 + 1 \) hyperbolas
Traces in \( y = k \), are
\( z^2 - x^2 = k^2 - 1 \) hyperbolas
Traces in \( z = k \), are
\( x^2 + y^2 = k^2 - 1 \) so circles
for \( |k| > 1 \)

(b) \( x^2 - y^2 - z^2 = 1 \) is still a hyperboloid of two sheets,
but the traces are circles along the \( x \)-axis instead of in the \( z \)-axis as in (a).