1. Determine whether each statement is true or false and circle your answer (2 points for correct answer, 1 for “I don’t know”, and 0 for wrong answer).

(a) The minimum value of the function \( f(x, y) = x^2 + y^2 \) subject to the constraint \( g(x, y) = x^4 + y^4 = 1 \) is 0.
   
   True \quad False \quad I don’t know

(b) If \( f(x, y) \) has two local maxima, then \( f \) must have a local minimum.

   True \quad False \quad I don’t know

(c) \( \int_0^1 \int_{x^2}^1 e^{x/y} dydx = \int_0^1 \int_0^{\sqrt{x}} e^{x/y} dydx \)

   True \quad False \quad I don’t know

2. The contours of \( f(x, y) = x^3 - 3xy + 3y^3 \) are shown on the right.

(a) Using the figure as an aid, find all critical points of \( f \).

\( f_x = 0 \Rightarrow 3x^2 - 3y = 0 \Rightarrow x^2 = y \)

\( f_y = 0 \Rightarrow 9y^2 - 3x = 0 \Rightarrow x = 3y^2 \)

so \( y(9y^2 - 1) = 0 \) so \( y = 0 \) or \( y = \frac{1}{3} \)

\( x = 0 \) or \( x = 3 \left( \frac{1}{3} \right)^{1/2} \text{C.R. at } (0,0) \) and \( \left( 3 \left( \frac{1}{3} \right)^{1/2} , \left( \frac{1}{3} \right)^{3/2} \right) \)

(b) Using the second derivative test, classify the critical points found in part (a).

\( f_{xx} = 6x \quad f_{xy} = 18y \quad f_{yx} = -3 \)

\( D(0,0) = (-3)^2 = -9 \quad (0,0) \text{ is a saddle.} \)

\( D \) at the other C.R. is positive, and \( f_{xx} > 0 \) so \( \left( 3 \left( \frac{1}{3} \right)^{1/2} , \left( \frac{1}{3} \right)^{3/2} \right) \) is a min.

(c) Doing any further work, at what point(s) in \([-1, 2] \times [-1, 2] \) does \( f \) attain the absolute maximum value? the absolute minimum value?

\text{Extra 3 points}

Looking at the contours, abs. max is at \( (-1, 2) \)

abs. min is at \( \left( 3 \left( \frac{1}{3} \right)^{1/2} , \left( \frac{1}{3} \right)^{3/2} \right) \).
3. Evaluate the following integral.

\[
\int_0^1 \int_0^{x^2} (x + 2y) \, dy \, dx = \int_0^1 (xy + y^2) \frac{x^2}{2} \, dx
\]

\[
= \int_0^1 \left[ x^3 + \frac{x^4}{2} \right] \, dx = \left[ \frac{x^4}{4} + \frac{x^5}{10} \right]_0^1 = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}.
\]

4. Set up the double integral that represents the volume of the solid bounded by the paraboloid \( z = x^2 + y^2 + 4 \) and the planes \( x = 0, y = 0, z = 0, \) and \( x + y = 1 \). Do not evaluate.

\[
\text{Volume} = \int_0^1 \int_0^{1-x} (x^2 + y^2 + 4) \, dy \, dx
\]

or

\[
\int_0^1 \int_0^{1-x} (x^2 + y^2 + 4) \, dx \, dy
\]

5. Rewrite the integral, \( \iint_D \sqrt{x^2 + y^2} \) where \( D \) is the region bounded by the semicircle \( x = \sqrt{4 - y^2} \) and the \( y \)-axis, in polar coordinates. Do not evaluate.

\[
\int_{-\pi/2}^{\pi/2} \int_0^2 \rho \cdot \rho \, d\rho \, d\theta
\]
6. Set up the integral to find the surface area of parametrized surface \( \mathbf{r}(u, v) = (uv, u+v, u-v) \), where \( u^2 + v^2 \leq 1 \), following the steps.

(a) Find the two partial derivatives \( \mathbf{r}_u \) and \( \mathbf{r}_v \).
\[
\mathbf{r}_u = \langle v, 1, 1 \rangle \quad \text{and} \quad \mathbf{r}_v = \langle u, 1, -1 \rangle
\]

(b) Find \( \mathbf{r}_u \times \mathbf{r}_v \), and \( |\mathbf{r}_u \times \mathbf{r}_v| \).
\[
\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u & v & 1 \\ 1 & 1 & -1 \end{vmatrix} = (-2) \mathbf{i} - (-v-u) \mathbf{j} + (v-u) \mathbf{k}
\]
\[
|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{4 + (v+u)^2 + (v-u)^2} = \sqrt{4 + 2v^2 + 2u^2} = \sqrt{4 + 2(v^2 + u^2)}.
\]

(c) Set up the integral in coordinate system of your choice. Do not evaluate.
\[
A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v| \, dA
\]

In \( uv \) coordinate system,
\[
A(S) = \int_{-1}^{1} \int_{\sqrt{1-v^2}}^{\sqrt{1-v^2}} \sqrt{4 + 2(v^2 + u^2)} \, du \, dv.
\]

In polar coordinates,
\[
A(S) = \int_{0}^{2\pi} \int_{0}^{1} r \sqrt{4 + 2r^2} \, r \, dr \, d\theta.
\]