Math 283 Quiz 4 Section 5: Oct 3

If you cannot complete a problem (perhaps because you forgot a formula) but you think you know how, please describe. Correct methods will receive partial credits.

1. Consider the parametric surface described by \( \mathbf{r}(u, v) = (u \cos v, u \sin v, v) \).
The grid-curves when \( u \) is a constant have which of the following shapes? (multiple-choice)
   \( \mathbf{r}'(u, v) = (u \cos v, u \sin v, 1) \)
   (a) lines
   (b) hyperbolas
   (c) parabolas
   (d) helixes

2. The arc length of \( \mathbf{r}(t) \) for \( a \leq t \leq b \) is given by \( \int_a^b |\mathbf{r}'(t)| \, dt \). Given \( \mathbf{r}(t) = t^2 \mathbf{i} + 2t \mathbf{j} + e^{-t} \mathbf{k}, \) set up (but not evaluate) the arc length integral for \( 0 \leq t \leq 1 \).
   \[
   |\mathbf{r}'(t)| = \sqrt{9t^4 + 4 + e^{-2t}}
   \]
   \[
   L = \int_0^1 \sqrt{9t^4 + 4 + e^{-2t}} \, dt
   \]

3. The position of a particle is given by \( \mathbf{r}(t) = (2t, t, e^t) \). Find
   (a) the velocity of the particle.
   \[
   \mathbf{v}(t) = \mathbf{r}'(t) = <2, 1, e^t>
   \]
   (b) the speed of the particle.
   \[
   \text{Speed} = |\mathbf{v}(t)| = \sqrt{2^2 + 1^2 + (e^t)^2} = \sqrt{5 + e^{2t}}
   \]