Math 283 Quiz 6 Section 5: Oct 17

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If you cannot complete a problem (perhaps because you forgot a formula) but you think you know how, please describe. Correct methods will receive partial credits.

1. We consider the surface \( z = \frac{y}{x} \) near the point \((1, 4)\).

   (a) What is the value of \( z \) at \((1, 4)\)?
   
   \[ z = \frac{4}{1} = 4. \]

   (b) Find the first order partial derivatives of \( z \) with respect to \( x \) and \( y \) (i.e. find \( z_x(1, 4) \) and \( z_y(1, 4) \)).
   
   \[ z_x(x, y) = y \cdot (-x^{-2}) = -\frac{y}{x^2}; \quad z_x(1, 4) = -\frac{4}{1} = -4. \]
   \[ z_y(x, y) = 1 \cdot \frac{1}{x}; \quad z_y(1, 4) = 1. \]

   (c) Write an equation of the tangent plane to \( z \) at \((1, 4)\).
   
   \[ z - 4 = (-4)(x - 1) + 1 \cdot (y - 4). \]

2. Use the chain rule to find \( \frac{dz}{dt} \).

   \[ z = \sqrt{2x - y}, \quad x = t^3, \quad y = 1 + 2t \]

   \[ \frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}. \]

   \[ \frac{\partial z}{\partial x} = \frac{1}{2} (2x - y)^{-\frac{1}{2}}, \quad 2 = \frac{1}{\sqrt{2x - y}}. \]

   \[ \frac{\partial z}{\partial y} = \frac{1}{2} (2x - y)^{-\frac{1}{2}}, \quad (-1) = -\frac{1}{\sqrt{2x - y}}. \]

   \[ \frac{dx}{dt} = 3t^2, \quad \frac{dy}{dt} = 2 \]

   \[ \frac{dz}{dt} = \frac{3t^2}{\sqrt{2x - y}} - \frac{2}{\sqrt{2x - y}}. \]