If you cannot complete a problem (perhaps because you forgot a formula) but you think you know how, please describe. Correct methods will receive partial credits.

1. Set up (but do not evaluate) the triple integral to find the volume of the solid bounded by the parabolic cylinder \( y = x^2 \) and the planes \( z = 0 \) and \( y + z = 1 \).

\[
\int_{-1}^{1} \int_{x^2}^{1-y} \int_{0}^{1-y} d z \, d y \, d x
\]

2. Rewrite (but do not evaluate) \( \iiint_{H} x^3 \sqrt{x^2 + y^2 + z^2} \, dV \) using spherical coordinates if \( H \) is the solid hemisphere centered at the origin with radius 1 above the xy-plane.

Integrand: \( x^2 + y^2 + z^2 = \rho^2 \) so \( x = \rho \sin \phi \cos \theta \).

Limits: \( x^2 + y^2 + z^2 \leq 1 \), so \( 0 \leq \rho \leq 1 \), \( 0 \leq \phi \leq \pi \), \( 0 \leq \theta \leq \pi \).

\[
\int_{0}^{2\pi} \int_{0}^{\pi} \int_{0}^{1} \rho^3 \sin^2 \phi \cos \phi \, d\rho \, d\phi \, d\theta
\]

\[
\rho^2 \sin \phi \, dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta
\]
3. Determine whether each statement is true or false and circle your answer (2 points for correct answer, 1 for “I don’t know”, and 0 for wrong answer).

(a) The curl of \( \mathbf{F}(x, y) = P(x, y)i + Q(x, y)j + 0k \) is a scalar multiple of the \( k \) vector.
- True
- False
- I don’t know

(b) If \( \mathbf{F}(x, y) \) is a vector field, then \( \nabla \times \mathbf{F} \) is a vector field.
- True
- False
- I don’t know

(c) If \( E \) is the solid enclosed by surface \( S \), then \( \iint_{S} (xz + yz + z^3) \cdot dS = \iiint_{E} (x + 2z) dV \)
by the divergence theorem.
- True
- False
- I don’t know

\[ \text{div} \left( <x, z, y, z, 2x> \right) = 1 + z + x = x + 2z \]

4. Consider a two dimensional vector field given by \( \mathbf{F}(x, y) = (3x^2 - 2xy + y^2)i + (-x^2 + 2xy)j \).

(a) Show that \( \mathbf{F}(x, y) \) above is a conservative vector field.

\[
\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 0 - 2x + 2y
\]

(b) Find a scalar valued function \( f(x, y) \) for which \( \nabla f(x, y) = \mathbf{F}(x, y) \) above.

\[
f = \int P \, dx = \int (3x^2 - 2xy + y^2) \, dx = x^3 - x^2 y + xy^2 + g(y)
\]

we have \( \frac{\partial g}{\partial y} = -x^2 + 2xy + g'(y) \) and we arrive to equation \(-x^2 + 2xy\).

so \( g'(y) = 0 \), \( g(y) = k \).

\[
f(x, y) = x^3 - x^2 y + xy^2 + k.
\]

(c) Using \( f(x, y) \) found in part (b), evaluate the line integral of \( \mathbf{F}(x, y) \) along \( C \) parametrized by \( \mathbf{r}(t) = (t + \sin \pi t)i + (2t + \cos \pi t)j \), \( 0 \leq t \leq 1 \) using the fundamental theorem of line integrals. (If you could not do part (b), use (the wrong) \( f(x, y) = x^2 + y^2 \)).

\[
\mathbf{r}'(0) = 0, \mathbf{r}'(1) = 1i + j
\]

\[
\int_{C} \mathbf{F} \cdot d\mathbf{r} = f(1, 1) - f(0, 1) = 1 + 1 + k - (k) = 1
\]
5. Consider the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) of a vector field \( \mathbf{F}(x, y) = \frac{x^2 + y^2}{P} \mathbf{i} + \frac{x}{Q} \mathbf{j} \) along some curve \( C \) parametrized by \( \mathbf{r}(t) \).

(a) If \( C \) is parametrized by \( \mathbf{r}(t) = t^3 \mathbf{i} + (t^2 + t) \mathbf{j} \) for \( 1 \leq t \leq 2 \), set up (but do not integrate) the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) along \( C \).

\[
\begin{align*}
\mathbf{F}' & = \langle 2t^3, 3t^2 + 2t + 1 \rangle, \\
\mathbf{F}' & = 6t^5 + (2t + 1)t^3 \sqrt{t^2 + t}
\end{align*}
\]

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F}' \, d\mathbf{r} = \int_1^2 (6t^5 + (2t + 1)t^3 \sqrt{t^2 + t}) \, dt
\]

(b) If \( C \) is parametrized by \( \mathbf{r}(t) = \cos t \mathbf{i} + 2\sin t \mathbf{j} \) for \( 0 \leq t \leq 2\pi \), set up (but do not integrate) the line integral using Green's Theorem.

\[
\begin{align*}
\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} & = 0, \\
\int_C \mathbf{F} \cdot d\mathbf{r} & = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) \, dA
\end{align*}
\]

6. Set up (but do not evaluate) the surface integral \( \iint_S xy \, dS \) where \( S \) is the part of the plane \( z = 1 - 2x - y \) that lies in the first octant (i.e. bounded by \( x = 0, y = 0, \) and \( z = 0 \)).

\[
\begin{align*}
\iint_S xy \, dS & = \iint_D \sqrt{(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 + 1} \, dA \\
& = \iint_D \frac{1}{\sqrt{5}} xy \, dA
\end{align*}
\]

- If you need to know your updated grade this weekend, please put your e-mail address here.

- I will hold a review session for the final on Friday, December 12th at 11am in AB 638 (the "conference" room or the "library" by the Math center).

- If you plan to take the final exam, it is at 9:45am-11:45am on Monday December 15th in AB634. If you have a conflict, please let me know in person or via e-mail, and we'll work out an alternative schedule.