Math 283: Quiz 9 (Take-home; due in class on Tues. Nov. 25)

Name: ______________________

Please use Maple, textbook, or notes if you like. You do not need to carry out the integrals here, unless otherwise specified. Note that there are 3 problems; 1 more on the back.

1. **Section 13.2 Problem 3**: Set up the line integral \( \int_C xy^4 \, ds \), where \( C \) is the right half of the circle \( x^2 + y^2 = 16 \).

2. We define \( \mathbf{F}(x, y) = (x^3 + 4xy) \mathbf{i} + (4xy - y^3) \mathbf{j} \). Answer the following questions regarding \( \mathbf{F} \).
   
   (a) Is \( \mathbf{F} \) conservative?

   (b) Set up the line integral of \( \mathbf{F} \) along the triangle whose vertices are (0,0), (1,0), and (0,1), traced counterclockwise. Note that this is a closed path. Can we say anything about the value of this integral without sketching the vector field?
3. We look at line integrals of the conservative vector field 
\[ \mathbf{G}(x, y) = \langle y, x \rangle. \]

(a) Sketch the vector field \( \mathbf{G} \) on the right.

(b) Set up the line integral \( \int_C \mathbf{G} \cdot d\mathbf{r} \) along \( C \) described by \( \mathbf{r}(t) = ti + t^2j \) for \( -1 \leq t \leq 2 \). Add \( C \) to your sketch in part (a). Do you expect that the value of the integral is positive, negative, or 0 from the sketch? Evaluate the integral by finding the scalar function \( g \) for which \( \nabla g = \mathbf{G} \).

(c) Set up the line integral \( \int_C \mathbf{G} \cdot d\mathbf{r} \) along \( C \) which is the arc of the full circle \( x^2 + y^2 = 16 \) traced counterclockwise. Do you expect that the value of the integral is positive, negative, or 0 from the sketch? Evaluate the integral.