1. Let $X_1, \ldots, X_{100}$ be the actual net weights of 100 randomly selected 50-lb bags of fertilizer. If the expected weight of each bag is 50 and the variance is 1, calculate an approximate probability

$$Pr(49.75 \leq \bar{X} \leq 50.25)$$

using the Central Limit Theorem.

2. The length of trout in Clear Lake has normal distribution with mean of 10.5 inches and a standard deviation of 2.1 inches. The length of trout in Blue Lake has normal distribution with mean of 9.6 inches and a standard deviation of 1.8 inches. John went fishing in Clear Lake and caught 14 trouts, Andy went fishing in Blue lake and caught 11 trouts. Find the probability that the mean length of John’s catch will exceed the mean length of Andy’s catch by at least 1 inch.

3. Let $X$ have a binomial distribution with $n = 4$ and an unknown probability of success $p$. In testing the null hypothesis $H_0 : p = 0.5$ against the alternative $H_1 : p = 0.25$, the critical region is $X \leq 1$. What is the probability of Type II error?

4. The time (in minutes) taken by a biological cell to divide into two cells has a normal distribution. From past experience, the standard deviation $\sigma$ can be assumed to be 3.5 minutes. When sixteen cells were observed, the mean time taken by them to divide was 31.2 minutes. Estimate the true mean time for a cell division using a 98 percent confidence interval.

5. An electrical firm that manufactures a certain type of bulb wants to estimate its mean life. Assuming that the life of the bulbs has a normal distribution with a standard deviation $\sigma = 40$ hours, find how many bulbs should be tested so as to be 90 percent confident that the sample mean will not differ from the true mean life by more than 10 hours.
6. The people representative claims that the true mean medical expenses during a year (for a family) are greater than $750. In a survey in which 100 randomly chosen middle-class families were interviewed, it was found that their mean medical expenses during a year were $770 with standard deviation of $120. Is the claim justified? Test appropriate hypothesis using the significance level $\alpha = 0.025$.

7. A ski coach claims that he can train beginning skiers for 3 weeks so that at the end of the program they will finish a certain downhill course in less than 13 minutes. It was found that, when a random sample of ten skiers was given the training, their mean time was 11.7 minutes with a standard deviation of $s = 1.2$ minutes. On the basis of this evidence, is the coach’s claim justified? Assume that the time to ski the course has normal distribution and test appropriate hypothesis using the significance level $\alpha = 0.05$.

8. A new vaccine is to be tested on the market. Find how large a sample should be drawn if we want to be 95 percent confident that the sample proportion will not differ from the true proportion by more than 0.02?

9. Consider the following sample of fat content (in percentage) of $n = 10$ randomly selected hot dogs:

   25.2  21.3  22.8  17.0  29.8  21.0  25.5  16.0  20.9  19.5

   Assuming that these data were selected from a normal population, derive
   (a) A 95% confidence interval for the population mean fat content.
   (b) A 95% prediction interval for the fat content of a single hot dog.

10. Let $p$ denote the proportion of all potential subscribers who favor cable company A over cable company B. Consider testing

    \[ H_0 : p = 0.5 \quad \text{versus} \quad H_1 : p > 0.5 \]

    based on a random sample of 9 individuals. Suppose that the null hypothesis is rejected if $X \geq 8$, where $X$ is the number of individuals in the sample who favor company A.
(a) Describe what Type I and Type II errors are in the context of this problem.

(b) What is the probability distribution of the test statistic \( X \) when \( H_0 \) is true? Use it to derive the probability of Type I error.

(c) Compute the probability of Type II error when \( p = .6 \). What is the power of the test for this value of \( p \)?

(d) What would you conclude if 5 of the 9 individuals in the sample favored company B?

11. Let \( \mu \) denote the true mean tread life of a certain type of tire. Consider a level \( \alpha = .05 \) test of

\[
H_0 : \mu = 20,000 \quad \text{versus} \quad H_1 : \mu > 20,000
\]

based on a sample of size \( n = 16 \) from a normal population with \( \sigma = 1500 \).

(a) Carry out the test if \( \bar{x} = 20,960 \). What is the probability of Type I error?

(b) What is the probability of Type II error if \( \mu = 21,000 \)? What is the power of the test for the same value of \( \mu \)?

(c) If it is required that the power of this test be 0.99 when \( \mu = 21,000 \), what sample size would be necessary?

12. Exam scores are normally distributed with the mean of 80 and the variance of 100.

(a) What is the probability of scoring 65 or less on the exam?

(b) What is the probability of scoring more than 90?

(c) Exams within the top 10% of the scores receive an A. What is the lowest score that would receive an A?

(d) Five students take the test in dependently. What is the chance that at most one of them scores below 65?

(e) If 100 students take the test, what is an approximate probability that more than 11 score below 65? [Hint: normal approximation to binomial]
13. An urn contains 5 white and 15 red balls.

(a) If 10 balls are randomly selected from the urn with replacement, find the probability that exactly 3 white are obtained.

(b) Balls are being selected from the urn with replacement until 3 white balls are obtained. What is the probability that 10 trials will be required?

(c) If 10 balls are randomly selected from the urn without replacement, find the probability that exactly 3 white are obtained.

14. When circuit boards used in the manufacture of compact disc players are tested, the long-run percentage of defectives is 5%. Let $X$ be the number of defective boards in a random sample of size $n = 25$.

(a) Determine $P(X \leq 2)$.

(b) Determine $P(1 \leq X \leq 4)$

(c) What is the probability that none of the 25 boards are defective?

(d) Determine the mean and the variance of $X$

15. Suppose the force acting on a column that helps to support a building is normally distributed with mean 15.0 kips and standard deviation 1.25 kips. What is the probability that the force

(a) Is at most 17 kips?

(b) Is between 10 and 12 kips?

(c) Differs from 15.0 kips by at most 2 standard deviations?