TYPES OF DISTRIBUTIONS

DISCRETE
C.d.f. is a step function

CONTINUOUS
C.d.f. is a continuous function

MIXED
C.d.f. neither step function nor continuous
**DISCRETE DISTRIBUTION**

1. The set of possible values of $X$ is either finite or countable.

   $$ A = \{ x_1, x_2, x_3, \ldots \} \quad \leftarrow \text{set of values of } X $$

   $$ P(X = x_i) = p_i > 0, \quad x_i \in A \quad \sum p_i = 1 $$

   $$ P_X(x) = P(X = x) = \begin{cases} p_i & \text{if } x = x_i, \ i = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases} $$

   the probability (mass) function of $X$ (p.m.f.)

   for any $A \subset \mathbb{R}$:

   $$ P(X \in A) = \sum_{x_i \in A} P(X = x_i) = \sum_{x_i \in A} P_X(x_i) $$

A discrete variable usually represents a discrete quantity (number of insurance claims in a given time period etc.)
**CONTINUOUS DISTRIBUTIONS**

A random variable $X$ has a continuous distribution if there exists a function $f: \mathbb{R} \to \mathbb{R}$ such that:

1. $f(x) \geq 0$ for all $x$
2. $\int_{-\infty}^{\infty} f(x) \, dx = 1$
3. For any interval $A$:
   $$P(X \in A) = \int_{A} f(x) \, dx$$

Continuous distributions are associated with continuous quantities (i.e., time to claim occurs, lifetime, rainfall, etc.)

**c.d.f.**:
$$F_X(x) = P(X \leq x) = P(X \in (-\infty, x]) = \int_{-\infty}^{x} f(t) \, dt$$

By the FTC:
$$F_X'(x) = f(x)$$

$f$ = probability density function (p.d.f.)

**NOTE**: If $X$ is continuous then $P(X = x) = 0$
Mixed Distributions

Distribution of $X$ is mixed if the c.d.f. of $X$ is neither continuous nor a step function.

$X$ = mixed r.v. with c.d.f. $F_X$

$F_X$ has jumps at $x_1, x_2, ...$ of size $k_1, k_2, ...$

Then:

$F_X(x) = (1-k)F(x) + \frac{k}{\Sigma k_j}G(x)$

$K = \Sigma k_j < 1$

$p_j = \frac{k_j}{k}$

$\Sigma p_j = 1$
Mixtures

\[ F_i < \text{cdf} \text{ for } i = 1, 2, 3, \ldots \]

\[ k_i - \text{satisfy } \sum_{i} k_i = 1, \quad k_i > 0 \]

\[ F(x) = \sum_{i} k_i F_i(x) \]

\( F \) is a discrete mixture of \( F_i \)'s with weights \( k_i \):

if cdf of \( X \) is \( F \) and

cdf of \( X_i \) is \( F_i \) then

\( X \) is a mixture of \( X_i \)'s

if \( F_p \) is a cdf for each \( p \in \mathbb{R} \)

\[ f \rightarrow \text{pdf} \]

\[ F(x) = \int_{-\infty}^{\infty} F_p(x) f(p) \, dp \]

\( \uparrow \) continuous mixture with weights given by \( f \)
25. An insurance policy pays for a random loss $X$ subject to a deductible of $C$, where $0 < C < 1$. The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 
2x & \text{for } 0 < x < 1 \\
0 & \text{otherwise.}
\end{cases}$$

Given a random loss $X$, the probability that the insurance payment is less than 0.5 is equal to 0.64.

Calculate $C$.

(A) 0.1
(B) 0.3
(C) 0.4
(D) 0.6
(E) 0.8