MATH 461/661 Test 1 Formula Sheet

1. Probability axioms:
   - For each $A \subset S$, $0 \leq P(A) \leq 1$.
   - $P(\bigcup A_i) = \sum P(A_i)$ whenever the events $A_i$ are mutually exclusive.
   - $P(S) = 1$.

2. Conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$ whenever $P(B) > 0$.

3. Independence: $A$ and $B$ are independent if $P(A \cap B) = P(A)P(B)$.

4. Useful probability rules:
   - If $A \subset B$ then $P(A) \leq P(B)$.
   - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
   - $P(\bigcup_{j=1}^{3} A_j) = \sum_{j=1}^{3} P(A_j) - \sum_{i<j} P(A_i \cap A_j) + P(A_1 \cap A_2 \cap A_3)$.
   - $P(A^c) = 1 - P(A)$; $P(A^c|B) = 1 - P(A|B)$
   - $P(A) = P(A \cap B) + P(A \cap B^c) = P(A|B)P(B) + P(A|B^c)P(B^c)$.
   - $P(A_1 A_2 \ldots A_n) = P(A_1)P(A_2|A_1)P(A_3|A_2 A_1) \cdots P(A_n|A_{n-1} \ldots A_1)$
   - For any events, $P(\bigcup A_i) \leq \sum P(A_i)$.

5. Total probability formula/Bayes rule: If $B_j$ are mutually exclusive and $\bigcup B_j = S$, i.e.
the sets $B_j$, $j = 1, 2, \ldots, n$ form a partition of the sample space, then for any $A$ we have
$P(A) = \sum P(A|B_j)P(B_j)$. In addition,

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum P(A|B_j)P(B_j)}.$$

6. If $X$ is a continuous (discrete) with pdf (pmf) $f$ then $P(X \in A) = \int_A f(x)dx$ (when $X$
is continuous) and $P(X \in A) = \sum_{x \in A} f(x)$ (when $X$ is discrete).

7. Cumulative distribution function (cdf): For any random variable $X$, the cdf of $X$ is the
function $F$ defined as $F(x) = P(X \leq x)$ for any $-\infty < x < \infty$. If $X$ is continuous,
then the pdf of $X$ is the derivative of the cdf of $X$. 
These formulas are meant to ease the burden of memorization for the second probability exam. However, it is your responsibility to review them and make sure that you understand the notation I used on this sheet. Enjoy!

1. **Expected value rules:**
   - $E(aX + b) = aE(X) + b$ for any constants $a$ and $b$.
   - **Mean of mixed distribution** If the CDF $F$ of $X$ jumps by $k_j$ at $x_j$, then
     \[
     E(X) = \int_{-\infty}^{\infty} xF'(x)dx + \sum_j x_jk_j.
     \]

2. **Variance properties.** The variance of $X$ is
   \[
   \sigma^2 = \text{Var}(X) = E[(X - EX)^2] = E(X^2) - [E(X)]^2.
   \]
   The standard deviation of $X$ is $\sigma = \sqrt{\text{Var}(X)}$.
   - $\text{Var}(X) = 0$ if and only if $P(X = C) = 1$ for some constant $C$.
   - $\text{Var}(aX + b) = a^2\text{Var}(X)$ for any constants $a$ and $b$.

3. **Moments.** The $k$th moment of $X$ is $E(X^k)$. The $k$th central moment of $X$ is $E[(X - EX)^k]$. The skewness and the kurtosis of $X$ are, respectively,
   \[
   \gamma_1 = \frac{E[(X - EX)^3]}{[\text{Var}(X)]^{3/2}}, \quad \gamma_2 = \frac{E[(X - EX)^4]}{[\text{Var}(X)]^2}.
   \]