PRACTICE MULTIPLE CHOICE TEST 1

1. On July 1, 1984 a person invested 1000 in a fund for which the force of interest at time \( t \) is given by \( \delta_t = \frac{3 + 2t}{50} \), where \( t \) is the number of years since January 1, 1984. Determine the accumulated value of the investment on January 1, 1985.

(A) 1036  (B) 1041  (C) 1045  (D) 1046  (E) 1051

2. Annuities X and Y provide the following payments:

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Annuity X</th>
<th>Annuity Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>1</td>
<td>( K )</td>
</tr>
<tr>
<td>11-20</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>21-30</td>
<td>1</td>
<td>( K )</td>
</tr>
</tbody>
</table>

Annuities X and Y have equal present values at an effective annual interest rate \( i \) such that \( v^{10} = \frac{1}{2} \). Determine \( K \).

(A) \( \frac{4}{3} \)  (B) \( \frac{3}{2} \)  (C) \( \frac{5}{3} \)  (D) \( \frac{7}{4} \)  (E) \( \frac{9}{5} \)

3. Deposits are to made to a fund each January 1 and July 1 for the years 1985 through 1995. The deposit made on each July 1 will be 10.25% greater than the one made on the immediately preceding January 1. The deposit made on each January 1 (except for January 1, 1985) will be the same amount as the deposit made on the immediately preceding July 1. The fund will be credited with interest at a nominal annual rate of 10%, compounded semi-annually. On December 31, 1995, the fund will have a balance of 11,000. Determine the initial deposit to the fund.

(A) 160  (B) 165  (C) 175  (D) 195  (E) 200

4. A 10,000 loan is to be repaid by 24 installments of 500 each, due at the end of the month. The loan is amortized by the direct ratio method (the “Rule of 78”). Compute the unpaid balance immediately following the sixth installment.

(A) 7500  (B) 7620  (C) 7740  (D) 7860  (E) 7890

5. A 1000 face value 20-year 8% bond with semi-annual coupons is purchased for 1014. The redemption value is 1000. The coupons are reinvested at a nominal annual rate of 6%, compounded semi-annually. Determine the purchaser’s annual effective yield rate over the 20-year period.

(A) 6.9%  (B) 7.0%  (C) 7.1%  (D) 7.2%  (E) 7.3%
6. A varying immediate annuity with a term of 2n years has a first payment equal to 1. Thereafter payments increase by 1 each year until they reach n at the end of n years. Payments remain at n for year n+1, and then decrease by 1 each year, with a final payment of 1 at the end of 2n years. Derive an expression for the present value of this annuity.

\[(A) \quad a_{\overline{n|}} \left( \frac{1}{d} - \frac{v^n}{i} \right) \quad \quad (B) \quad a_{\overline{n|}} \left( \frac{1}{d} + \frac{v^n}{i} \right) \quad \quad (C) \quad a_{\overline{n|}} \left( \frac{1}{d} - \frac{v^n}{d} \right) \quad \quad (D) \quad a_{\overline{n|}} \left( \frac{1}{d} + \frac{v^n}{d} \right) \quad \quad (E) \quad a_{\overline{n|}} \left( \frac{1}{d} + \frac{v^n}{d} \right)\]

7. You are given that \(a(t) = Kt^2 + Lt + M\), for \(0 \leq t \leq 2\), and that \(a(0) = 100\), \(a(1) = 110\), and \(a(2) = 136\). Determine the force of interest at time \(t = \frac{1}{2}\).

(A) .030 \quad \quad (B) .049 \quad \quad (C) .061 \quad \quad (D) .095 \quad \quad (E) .097

8. A loan is being repaid by 15 annual installments of 1000 each. Interest is at an effective annual rate of 5%. Immediately after the fifth installment is paid the loan is renegotiated. The revised amortization schedule calls for a sixth payment of 800, a seventh installment of \((800 + K)\), with each subsequent installment increasing by K over the previous payment. The period of the loan is not changed. Determine the revised amount of the last installment.

(A) 1240 \quad \quad (B) 1290 \quad \quad (C) 1360 \quad \quad (D) 1440 \quad \quad (E) 1460

9. An asset has an initial value of 1000 and a salvage value of 100 at the end of 10 years. Ten-year depreciating schedules are determined under the following four methods:

I. Compound-interest at 5% /  
II. Straight-line  
III. Constant percentage  
IV. Sum-of-the-digits

Rank the first-year depreciation under the four methods.

(A) I < II < III < IV \quad \quad (B) I < II < IV < III \quad \quad (C) I < III < II < IV \quad \quad (D) II < I < IV < III \quad \quad (E) III < I < II < IV

10. You are given the following values at an effective annual interest rate of 10%:

\((1a)_{\overline{n|}} = 55.00\) and \(a_{\overline{n|}} = 8.08\).

Using \(\frac{\partial a_{\overline{n|}}}{\partial t}\), calculate an approximate value of \(a_{\overline{n|}}\) at an effective annual rate of 10.20%.

(A) 7.92 \quad \quad (B) 7.95 \quad \quad (C) 7.98 \quad \quad (D) 8.01 \quad \quad (E) 8.04

11. A 9% bond with a 1000 par value and coupons payable semi-annually is redeemable at maturity for 1100. At a purchase price of \(P\), the bond yields a nominal annual interest rate of 8%, compounded semi-annually, and the present value of the redemption amount is 190. Determine \(P\).

(A) 1050 \quad \quad (B) 1085 \quad \quad (C) 1120 \quad \quad (D) 1165 \quad \quad (E) 1215
12. A person deposits 100 at the beginning of each year for 20 years. Simple interest at an annual rate of \( i \) is credited to each deposit from the date of deposit to the end of the twenty-year period. The total amount thus accumulated is 2840. If instead, compound interest had been credited at an effective annual rate of \( i \), what would the accumulated value of these deposits have been at the end of twenty years?

\[
\begin{array}{ccc}
(A) \ 2980 & (B) \ 3100 & (C) \ 3200 \\
(D) \ 3310 & (E) \ 3470 \\
\end{array}
\]

13. On January 1, 1985, Marc has the following options for repaying a loan:

(i) Sixty monthly payments of 100 beginning February 1, 1985.
(ii) A single payment of 6000 at the end of \( K \) months.

Interest is at a nominal annual rate of 12% compounded monthly. The two options have the same present value. Determine \( K \).

\[
\begin{array}{ccc}
(A) \ 29.0 & (B) \ 29.5 & (C) \ 30.0 \\
(D) \ 30.5 & (E) \ 31.0 \\
\end{array}
\]

14. A ten-year adjustable rate mortgage loan of 23,115 is being repaid with quarterly installments of 1000 based upon an initial interest rate of 12% compounded quarterly. Immediately after the twelfth payment, the interest rate is increased to 14% compounded quarterly. The quarterly installments remain at 1000. Calculate the loan balance immediately after the 24th payment.

\[
\begin{array}{ccc}
(A) \ 12,000 & (B) \ 12,550 & (C) \ 12,950 \\
(D) \ 13,350 & (E) \ 13,750 \\
\end{array}
\]

15. The present value of 200 paid at the end of \( n \) years, plus the present value of 100 paid at the end of \( 2n \) years is 200. Determine the annual effective rate of interest.

\[
\begin{array}{ccc}
(A) \ \left( \frac{\sqrt{3}+1}{2} \right)^{\frac{1}{n}} - 1 & (B) \ 1 - \left( \frac{\sqrt{3}-1}{2} \right)^{\frac{1}{n}} & (C) \ \left( \frac{\sqrt{3}-1}{2} \right)^{\frac{1}{n}} - 1 \\
(D) \ \left( \frac{1+\sqrt{3}}{2} \right)^{\frac{1}{2n}} - 1 & (E) \ 1 - \left( \frac{\sqrt{3}-1}{2} \right)^{\frac{1}{2n}} \\
\end{array}
\]

16. The duration of a bond at interest rate \( i \) is defined as

\[
\left( \sum t \cdot C_t \cdot v^t \right) \div \left( \sum C_t \cdot v^t \right),
\]

where \( C_t \) represents the net cash flow from the coupons and the maturity value of the bond at time \( t \). You are given a 1000 par value 20-year bond with 4% annual coupons and a maturity value of 1000. Calculate the duration of this bond at 5% interest.

\[
\begin{array}{ccc}
(A) \ 5.5 & (B) \ 8.9 & (C) \ 13.7 \\
(D) \ 20.0 & (E) \ 24.0 \\
\end{array}
\]
17. A company buys two machines. Both machines are expected to last 14 years, and each has a scrap value of 1050. Machine A costs 2450, whereas Machine B costs \( Y \). The depreciation method used for Machine A is the straight-line method, whereas the depreciation method used for Machine B is the sum-of-the-digits method. The present value of the depreciation charges made at the end of each year for Machines A and B are equal. Given that \( i = .10 \), calculate \( Y \).

(A) 2110  (B) 2200  (C) 2220  (D) 2320  (E) 2340

18. You are given the following information on a bond:

(i) Par value = 1000.
(ii) Redemption Value = 1000.
(iii) Coupon rate = 12%, convertible semi-annually.
(iv) It is priced to yield 10%, convertible semi-annually.

The bond has a term of \( n \) years. If the term of the bond is doubled, the price will increase by 50. Calculate the price of the \( n \)-year bond.

(A) 1050  (B) 1100  (C) 1150  (D) 1200  (E) 1250

19. Given that \( a_{71} = K, a_{111} = L, \) and \( a_{181} = M \), find an expression for \( i \).

(A) \( \frac{L \cdot K}{L + K + M} \)  (B) \( \frac{L \cdot K}{L + K - M} \)  (C) \( \frac{L + K + M}{L \cdot K} \)  (D) \( \frac{L + K - M}{L \cdot K} \)  (E) None of A,B,C,D

20. An investor purchased a 5-year financial instrument having the following features:

(i) The investor receives payments of 1000 at the end of each year for 5 years.

(ii) These payments earn interest at an effective rate of 4% per year. At the end of the year, this interest is reinvested at the rate of 3% per year.

Calculate the purchase price to the investor to produce a yield rate of 4%.

(A) 4450  (B) 4580  (C) 4620  (D) 4690  (E) 4760
PRACTICE MULTIPLE CHOICE TEST 2

1. At a certain interest rate the present values of the following two payment patterns are equal:

   (i) 200 at the end of 5 years plus 500 at the end of 10 years;
   (ii) 400.94 at the end of 5 years.

   At the same interest rate, 100 invested now plus 120 invested at the end of 5 years will accumulate to P at the end of 10 years. Find P.

   (A) 910  (B) 918  (C) 942  (D) 967  (E) 992

2. On January 1, 1987, three 100 par value bonds with 6% annual coupons will mature at the end of 1, 2 and 3 years, respectively. The redemption value of each bond is 100. You are given that the prices for these bonds on January 1, 1987 are as follows:

   \[
   \begin{array}{|c|c|}
   \hline
   \text{Maturity Date} & \text{Price} \\
   \hline
   \text{December 31, 1987} & 101.92 \\
   \text{December 31, 1988} & 102.84 \\
   \text{December 31, 1989} & 105.51 \\
   \hline
   \end{array}
   \]

   These prices are based on an interest rate of \(i\) in 1987, \(j\) in 1988, and \(k\) in 1989. Determine \(j\).

   (A) 2\%  (B) 3\%  (C) 4\%  (D) 5\%  (E) 6\%

3. You are given the following values:

   (i) \(\frac{a_{10}}{20} = 11.196\)
   (ii) \((1a)_{20} = 95.360\)

   Calculate \(i\).

   (A) .060  (B) .063  (C) .065  (D) .068  (E) .070

4. An investment of 1 will double in 27.72 years at a force of interest \(\delta\). An investment of 1 will increase to 7.04 in \(n\) years at a nominal rate of interest numerically equal to \(\delta\) and convertible once every two years. Calculate \(n\).

   (A) 78  (B) 79  (C) 80  (D) 81  (E) 82

5. Fund A accumulates at a rate of 12\% convertible monthly. Fund B accumulates with a force of interest \(\delta_t = \frac{t}{6}\), for all \(t\). At time \(t = 0\), 1 is deposited in each fund. \(T\) is the time that the two funds are equal, \(T > 0\). Determine \(T\).

   (A) 12 \cdot \ln(1.01)  (B) 12 \cdot [\ln(1.12) - \ln(1.01)]  (C) 12 \cdot \ln(1.12)
   (D) 144 \cdot \ln(1.01)  (E) 144 \cdot \ln(1.12)
6. A new machine which costs 11,000 has a scrap value of 900. The machine has a useful lifetime of 100 years.

\[ BVSL_t \] is the book value of the asset at the end of year \( t \) under the straight-line depreciation method.

\[ BVSD_t \] is the book value of the asset at the end of year \( t \) under the sum-of-the-digits depreciation method.

For what value of \( t \) is \( (BVSD_t - BVSL_t) \) a minimum?

(A) 20  (B) 30  (C) 40  (D) 50  (E) 60

7. A 100,000 loan is to be repaid by 30 equal payments at the end of each year. The outstanding balance is amortized at 4%. In addition to the annual payments, the borrower must pay an origination fee at the time the loan is made. This fee is 2% of the loan but does not reduce the loan balance. Then the second payment is due, the borrower pays the remaining loan balance. Determine the yield to the lender considering the origination fee and the early pay-off of the loan.

(A) 4.9%  (B) 5.0%  (C) 5.1%  (D) 5.2%  (E) 5.3%

8. A 100 par value 100-year bond with a redemption value of 100 has annual coupons of 10% for the first 10 years, 9% for the next 10 years, 8% for the next 10 years, ..., 1% for the last 10 years. Calculate the price of the bond to yield \( i \).

(A) \[ \frac{10s_{10|} - s_{10|}}{s_{10|}} + 100v^{100} \]  (B) \[ \frac{10s_{10|} - a_{10|}}{i \cdot s_{10|}} + 100v^{100} \]

(C) \[ \frac{10s_{10|} - v^{10}a_{10|}}{s_{10|}} + 100v^{100} \]  (D) \[ \frac{10s_{10|} - a_{100|}}{s_{10|}} + 100v^{100} \]

(E) \[ \frac{10s_{10|} - a_{100|}}{i \cdot s_{10|}} + 100v^{100} \]

9. A perpetuity pays 1 at the end of every year plus an additional 1 at the end of every second year. The present value of the perpetuity is \( K \) for \( i \geq 0 \). Determine \( K \).

(A) \[ \frac{i + 3}{i(i+2)} \]  (B) \[ \frac{i + 2}{i(i+1)} \]  (C) \[ \frac{i + 1}{i^2} \]  (D) \[ \frac{3}{2i} \]  (E) \[ \frac{i + 1}{i(i+2)} \]

10. Fund A accumulates at a force of interest of \( \delta_t = a + bt \). Fund B accumulates at a force of interest of \( \delta_t = g + ht \). Fund A equals Fund B at \( t = 0 \) and at \( t = n \). Given that \( a > g > 0 \) and \( h > b > 0 \), determine \( n \).

(A) \[ \frac{a - g}{h - b} \]  (B) \[ \frac{2(a - g)}{h - b} \]  (C) \[ \frac{h - b}{a - g} \]  (D) \[ \frac{h - b}{2(a - g)} \]  (E) \[ \frac{2(h - b)}{a - g} \]
11. An annuity pays 1 at the end of each 4-year period for 40 years. Given $a_{84} = k$, find the present value of the annuity.

(A) $\frac{1 - (1 - ik)^5}{1 - (1 - ik)^{35}}$  (B) $\frac{1 - (1 - ik)^{40}}{1 - (1 - ik)^5}$  (C) $\frac{1 - (1 - ik)^5}{i}$  (D) $\frac{1 - (1 - ik)^{40}}{(1 - ik)^{-5} - 1}$  (E) $\frac{1 - (1 - ik)^5}{(1 - ik)^{-5} - 1}$

12. An investment fund has a value of 1000 at the beginning and the end of the year. A deposit of 200 was made at the end of four months. A withdrawal of 300 was made at the end of seven months. Find the rate of interest earned by the fund assuming simple interest during the year.

(A) .099  (B) .100  (C) .105  (D) .108  (E) .111

13. A company has a lease expiring on December 31, 1986. The company is notified that the monthly rate will double as of January 1, 1987. This rate will be good for two years. The company wishes to dampen the effect of the rent increase by paying a higher rent for 2 1/2 years, starting July 1, 1986. Calculate the percentage increase on July 1, 1986 assuming an interest rate of 12% compounded monthly.

(A) 70%  (B) 72%  (C) 74%  (D) 76%  (E) 78%

14. Fund F accumulates at rate $\delta = \frac{1}{1 + t}$. Fund G accumulates at rate $\delta = \frac{4t}{1 + 2t^2}$. $F(t)$ is the amount in Fund F at time $t$, and $G(t)$ is the amount in Fund G at time $t$, with $F(0) = G(0)$. Let $H(t) = F(t) - G(t)$. Calculate $T$, the value of time $t$ when $H(t)$ is a maximum.

(A) $\frac{1}{4}$  (B) $\frac{1}{2}$  (C) $\frac{3}{4}$  (D) 1  (E) $\frac{5}{4}$

15. A 35-year loan is to be repaid in equal annual installments. The amount on interest paid in the 8th installment is 135. The amount of interest paid in the 22nd installment is 108. Calculate the amount of interest paid in the 29th installment.

(A) 72  (B) 73  (C) 74  (D) 75  (E) 76

16. A deposit of 1 will accumulate to 2.7183 in 10 years at force of interest given by

$\delta_t = \begin{cases} \frac{kt}{10}, & 0 < t \leq 5 \\ \frac{kt^2}{20}, & 5 < t \leq 10 \end{cases}$

Calculate $k$.

(A) .01  (B) .02  (C) .03  (D) .04  (E) .05
17. A common stock pays annual dividends at the end of each year. The earnings per share in the year just ended were \( J \). Earnings are assumed to grow 10% per year in the future. The percentage of earnings paid out as a dividend will be 0% for the next 5 years and 50% thereafter. Calculate the theoretical price of the stock to yield the investor 21%.

(A) \( \frac{5J}{(1.10)^4} \)  
(B) \( \frac{5J}{(1.10)^5} \)  
(C) \( \frac{5J}{(1.10)^6} \)  
(D) \( \frac{10J}{(1.10)^5} \)  
(E) \( \frac{10J}{(1.10)^6} \)

18. A company buys two assets. Both assets cost 1800, have the same scrap value, and are to be depreciated over a 15-year period. One asset is depreciated using the straight-line method. The second one is depreciated using the sinking fund method. Under this method, \( i = 5\% \), and the depreciation charge for the \( 7^{th} \) year equals 100. What is the sum of the depreciation charges on both assets for year 12?

(A) 229  
(B) 230  
(C) 235  
(D) 241  
(E) 248

19. A sum, \( P \), is used to buy a deferred perpetuity-due of 1 payable annually. The effective annual rate of interest is \( i \), \( i > 0 \). Calculate the deferred period.

(A) \( \log\left(\frac{P}{d}\right) \)  
(B) \( 1 - \frac{\log(iP)}{\delta} \)  
(C) \( -\frac{\log(iP)}{\delta} \)  
(D) \( 1 + \frac{\log(dP)}{\delta} \)  
(E) \( \frac{\log(dP)}{\delta} \)

20. An investment of 700 is to be used to make payments of 10 at the end of the first year, 20 at the end of the second year, 30 at the end of the third year, and so on, every year for as long as possible. A smaller final payment is paid one year after the last regular payment. The fund earns an effective rate of interest of 5%. Calculate the amount of the smaller final payment.

(A) 35  
(B) 67  
(C) 70  
(D) 74  
(E) 146
1. A loan of 1000 is made at an interest rate of 12% compounded quarterly. The loan is to be repaid with three payments: 400 at the end of the first year, 800 at the end of the fifth year, and the balance at the end of the tenth year. Calculate the amount of the final payment.

(A) 587  (B) 658  (C) 737  (D) 777  (E) 812

2. On January 1, 1986, Sam invests 1000 in a fund for which the force of interest at time $t$ is expressed by $.10(t - 1)^2$, where $t$ is the number of years since January 1, 1986. Calculate the accumulated value of the fund on January 1, 1988.

(A) 1065  (B) 1067  (C) 1069  (D) 1071  (E) 1073

3. You are given $\delta_t = \frac{2}{10 + t} - t \geq 0$. Calculate $a_{\frac{4}{3}}$.

(A) 2.34  (B) 2.62  (C) 2.85  (D) 3.01  (E) 3.23

4. A car dealer offers to sell a car for 10,000. The current market loan rate is a nominal rate of interest of 12% per annum, compounded monthly. As an inducement, the dealer offers 100% financing at an effective annual interest rate of 5%. The loan is to be repaid in equal installments at the end of each month over a four-year period. Calculate the cost to the dealer of this inducement.

(A) 700  (B) 900  (C) 1100  (D) 1300  (E) 1500

5. A 30-year bond has an annual coupon rate of 6% for the first 10 years, 7% for the next 10 years, 8% for the last 10 years, and matures at its par value of 100. The bond is bought to produce an effective annual yield rate of 7%. Determine an expression for the price of the bond. (All interest functions are at 7%).

(A) $6a_{\frac{10}{10}} + 7v^{10}a_{\frac{10}{10}} + 8v^{20}a_{\frac{10}{10}} + \frac{100}{(1.06)(1.07)(1.08)}$  (D) $7a_{\frac{30}{30}} + 100v^{30}$

(B) 100  (E) $100 + v^{20}a_{\frac{10}{10}} + a_{\frac{10}{10}}$

(C) $100 - v^{20}a_{\frac{10}{10}} + a_{\frac{10}{10}}$

6. A building with an original cost of 1,000,000 and an asset life of 60 years is depreciated using the constant-percentage method. After four years, the book value of the building is 918,820. Calculate the salvage value of the building.

(A) 258,000  (B) 280,800  (C) 365,100  (D) 508,000  (E) 529,900
7. Determine the present value of 1 payable at the end of years 7, 11, 15, 19, 23, and 27.
   
   (A) \[ \frac{a_{28} - a_{4}}{s_{4}} \]  
   (B) \[ \frac{a_{28} - a_{4}}{s_{4}} \]  
   (C) \[ \frac{a_{28} - a_{4}}{s_{3} + a_{1}} \]  
   (D) \[ \frac{a_{28} - a_{4}}{s_{3} - a_{1}} \]  
   (E) \[ \frac{a_{28} - a_{4}}{s_{3} + a_{1}} \]  

8. You are given the following information:
   
   (i) The sum of the present values of a payment of \( X \) at the end of 10 years and a payment of \( Y \) at the end of 20 years is equal to the present value of a payment of \( X + Y \) at the end of 15 years.
   (ii) \( X + Y = 100 \)
   (iii) \( i = 5\% \)

   Calculate \( X \).
   
   (A) 44  
   (B) 48  
   (C) 50  
   (D) 52  
   (E) 57

9. Janis needs an amount on January 1, 2025 to provide for a lump sum of 50,000 and a 15-year annuity-due with semi-annual payments of \( K \). The amount will be accumulated by 25 annual deposits of \( K \) beginning on January 1, 2000. The deposits accumulate at a nominal rate of 4\% compounded semi-annually. The annuity payout is based on a nominal rate of 3\% compounded semi-annually. Determine an expression for \( K \).
   
   (A) \[ \frac{50,000}{s_{50/102} - a_{30/1015}} \]  
   (B) \[ \frac{50,000}{s_{50/102} - a_{20/1015}} \]  
   (C) \[ \frac{50,000}{s_{20/102} - a_{30/1015}} \]  
   (D) \[ \frac{50,000}{s_{20/102} - a_{30/1015}} \]  
   (E) \[ \frac{50,000}{s_{20/102} - a_{30/1015}} \]  

10. A bond with coupons equal to 40 sells for \( P \). A second bond with the same maturity value and term has coupons equal to 30 and sells for \( Q \). A third bond with the same maturity value and term has coupons equal to 80. All prices are based on the same yield rate, and all coupons are paid at the same frequency. Determine the price of the third bond.
   
   (A) \( 4P - 4Q \)  
   (B) \( 4P + 4Q \)  
   (C) \( 4Q - 3P \)  
   (D) \( 5P - 4Q \)  
   (E) \( 5Q - 4P \)  

11. A continuously increasing annuity with a term of \( n \) years has payments payable at an annual rate \( t \) at time \( t \). The force of interest is equal to \( \frac{1}{n} \). Calculate the present value of this annuity.
   
   (A) \( n^{2}(1 - e^{-1/n}) \)  
   (B) \( n^{2}(1 - 2e^{-1/n}) \)  
   (C) \( n^{2}(1 - e^{-1}) \)  
   (D) \( n^{2}(1 - 2e^{-1}) \)  
   (E) \( n^{2}(1 - e^{-n}) \)
12. A car loan of 10,000 will be repaid by thirty monthly payments of 387.48. The loan is amortized by the direct ratio method. Calculate the interest contained in the twentieth payment.

(A) 38.43   (B) 38.86   (C) 39.30   (D) 39.74   (E) 40.17

13. A loan of 125,000 will be repaid by payments at the end of each month over 30 years. Payments for a given year are level and are 2% greater than those for the previous year. The monthly payment for the first year is P. The effective annual interest rate is 5%. Calculate P.

(A) 516   (B) 526   (C) 537   (D) 547   (E) 558

14. You are given:

(i) \( \delta_t = \frac{2t^2 + 8t}{t^4 + 8t^2 + 16} \), \( 0 \leq t \leq 1 \)

(ii) i is the effective annual rate equivalent to \( \delta_t \).

(iii) Fund X accumulates with simple interest at rate i.

(iv) Fund Y accumulates at \( \delta_t \).

(v) An amount of 1 is deposited in each fund at time \( t = 0 \).

At what time, \( t \), is the excess of Fund X over Fund Y a maximum?

(A) .250   (B) .375   (C) .500   (D) .625   (E) .750

15. As settlement of a 100,000 death benefit, a beneficiary elected to take an annuity-immediate payable monthly for 25 years. The monthly payment was calculated using an effective annual interest rate of 3%. After making payments for 10 years, the insurance company decides to increase the monthly payments for the remaining 15 years by changing the effective annual interest rate to 5%. Calculate the increase in the monthly payment.

(A) 60   (B) 62   (C) 64   (D) 66   (E) 68

16. James invests 2000 at an effective annual interest rate of 17% for 10 years. Interest is payable annually and is reinvested at effective annual rate \( j \). At the end of 10 years, James' accumulated interest is \( k \). Peter invests 150 at the end of each year for 20 years at effective annual rate 14%. Interest is paid annually and reinvested at effective annual rate \( j \). Find an expression for Peter's accumulated interest after 20 years. (In answers, \( I = \frac{jk}{340} \))

(A) \( \frac{21}{j} \left[ \frac{(1+j)^2 - 1}{j} \right] - 20 \)   (B) \( \frac{21}{j} \left[ \frac{(1+j)^2 - 1}{j(1+j)^2} \right] - 20 \)   (C) \( \frac{21}{j} \left[ \frac{(1+j)^2 - 1}{j} \right] - 19 \)   (D) \( \frac{21}{j} \left[ \frac{(1+j)^2 - 20}{j} \right] \)   (E) None of A, B, C, D
17. Which of the following are equal to 17?

I. \( \frac{a_{10|}(1 + i \cdot s_{10|})}{1 + \bar{s}_{9|}} \)
II. \( v^{10} \cdot \bar{s}_{10|} - a_{9|} \)
III. \( (1+i)^{10} \cdot a_{10|} - \bar{s}_{9|} \)

(A) I & II  (B) I & III  (C) II & III  (D) All  (E) None of A, B, C, D

18. A perpetuity has payments at the end of each four-year period. The first payment at the end of four years is 1. Each subsequent payment is 5 more than the previous payment. Calculate the present value of this perpetuity, given that \( v^4 = .75 \).

(A) 45  (B) 48  (C) 52  (D) 60  (E) 80

19. A 1000 par value, 8% bond with quarterly coupons is callable five years after issue. The bond matures for 1000 at the end of ten years, and is sold to yield a nominal rate of 6% compounded quarterly under the assumption that the bond will not be called. Calculate the redemption value, at the end of five years, that will yield the purchaser the same nominal rate of 6% compounded quarterly.

(A) 1060  (B) 1067  (C) 1073  (D) 1080  (E) 1086

20. A bank offers the following certificates of deposit:

<table>
<thead>
<tr>
<th>Term in years</th>
<th>Nominal annual interest rate (compounded semi-annually)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5%</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

The bank requires that interest accumulate at the certificate’s interest rate, and does not permit early withdrawal. The certificates mature at the end of the term. During the next six years the bank will continue to offer these certificates of deposit. Jeff invests 1000 in the bank. Calculate the maximum amount he can withdraw at the end of six years.

(A) 1480  (B) 1510  (C) 1540  (D) 1570  (E) 1600
PRACTICE MULTIPLE CHOICE TEST 4

1. You are given \( \int_0^n \frac{1}{t+1} \, dt = 100 \). Calculate \( \frac{n}{n+1} \).
   (A) 100n\delta  \quad (B) n\delta  \quad (C) n - 100\delta  \quad (D) 100 - n\delta  \quad (E) n - \frac{\delta}{100}

2. In Fund A, the accumulated value of 1 at any time \( t > 0 \) is \( 1 + t \). In Fund B, the accumulated value of 1 at any time \( t > 0 \) is \( 1 + t^2 \). \( T \) is the time when the force of interest for Fund A is equal to the force of interest for Fund B. Calculate \( T \).
   (A) .41 \quad (B) 1.00 \quad (C) 1.41 \quad (D) 2.00 \quad (E) 2.11

3. Sue loans Betty an amount \( L \), which is repaid by 30 equal annual payments on an amortization basis at rate \( j \). Sue remits each payment to a fund earning 6%. The accumulated value of the fund immediately following the final deposit is \( k \). Find an expression for \( L \), given that \( s_{30j} = m \).
   (A) \( \frac{79.058(1 + jm)}{km} \) \quad (B) \( \frac{79.058(1 + j)}{km} \) \quad (C) \( \frac{km}{1 + jm} \) \quad (D) \( \frac{km}{79.058(1 + jm)} \) \quad (E) None of A, B, C, D

4. A loan, for amount \( A \), is to be amortized by \( n \) annual payments of 1, based on an interest rate of \( i \). \( P \) is the present value, at interest rate \( i \), of the principal portions of the loan payments. Determine an expression for \( (\frac{A}{n})^{\frac{1}{n}} \).
   (A) \( \frac{v^2}{i} (A - P) \) \quad (B) \( \frac{v}{i} (A - P) \) \quad (C) \( \frac{1}{i} (A - P) \) \quad (D) \( \frac{1 + i}{d} (A - P) \) \quad (E) \( \frac{d}{i} (A - P) \)

5. An insurance company owns a 1000 par value 10% bond with semiannual coupons. The bond will mature for 1000 at the end of 10 years. The company decides that an 8-year bond would be preferable. Current yield rates are 7% compounded semiannually. The company uses the proceeds from the sale of the 10% bond to purchase a 6% bond with semiannual coupons, maturing at par at the end of 8 years. Calculate the par value of the 8-year bond.
   (A) 1000 \quad (B) 1291 \quad (C) 1306 \quad (D) 1419 \quad (E) 1497

6. In Fund X money accumulates at force of interest \( \delta_t = .01t + .10 \), for \( 0 \leq t \leq 20 \). In Fund Y money accumulates at annual effective rate \( i \). An amount of 1 is invested in each of Fund X and Fund Y for 20 years. The value of Fund X at the end of 20 years is equal to the value of Fund Y at the end of 15 years. Calculate the value of Fund Y at the end of 15 years.
   (A) \( e^{10} \) \quad (B) \( e^{20} \) \quad (C) \( e^{-30} \) \quad (D) \( e^{-40} \) \quad (E) \( e^{50} \)

7. A loan of 10,000 at interest rate 12% per annum is repaid with four payments: (i) 1000 at the end of 3 months; (ii) 2000 at the end of 6 months; (iii) 3000 at the end of 9 months; (iv) \( X \) at the end of 12 months. Determine \( X \), using the United States Rule.
   (A) 4780 \quad (B) 4789 \quad (C) 4900 \quad (D) 4908 \quad (E) 4951
8. The present value of a payment of 1004 at the end of $T$ months is equal to the present value of 314 after 1 month, 271 after 18 months, and 419 after 24 months. The effective annual interest rate is 5%. Calculate $T$.

(A) 14   (B) 15   (C) 16   (D) 17   (E) 18

9. Five 1000 par value bonds are purchased to yield 8% compounded semiannually. The redemption value of each bond is 1020. Each bond has annual coupons at 5%. One bond matures at the end of each of years 6 through 10. Calculate the total price of the five bonds.

(A) 4158   (B) 4186   (C) 4198   (D) 4201   (E) 4227

10. A perpetuity pays 1 at the end of the first year, 2 at the end of the second year, 3 at the end of the third year, and so on. Which of the following expressions give the present value of this perpetuity?

I. \( \frac{1}{i^2} \)  
II. \( \frac{1-d}{d^2} \)  
III. \( e^\delta + e^{-\delta} \)

(A) I & II   (B) I & III   (C) II & III   (D) II only   (E) None of A, B, C, D

11. The proceeds of a life insurance policy are left on deposit, with interest credited at the end of each year. The beneficiary makes withdrawals from the fund at the end of each year $t$, $t = 1, 2, \ldots, 10$. At the minimum interest rate of 3% guaranteed in the policy, the equal annual withdrawal would be 1000. However the insurer credits interest at 4% for the first four years and 5% for the next six years. The actual amount withdrawn at the end of the year $t$ is $W_t = \frac{F_t}{\delta_{11-t} | .03}$, where $F_t$ is the amount of the fund, including interest, prior to the withdrawal. Calculate $W_{10}$.

(A) 1160   (B) 1167   (C) 1172   (D) 1177   (E) 1183

12. You are given $\delta_t = \frac{2}{1 + t}$. A payment of 300 at the end of 3 years and 600 at the end of 6 years has the same present value as a payment of 200 at the end of 2 years and $X$ at the end of 5 years. Calculate $X$.

(A) 306   (B) 316   (C) 376   (D) 456   (E) 506
13. X, Y, and Z are the purchase prices of three assets to be depreciated over a 10-year period. The salvage value of each asset is 100. You are given:

<table>
<thead>
<tr>
<th>Price</th>
<th>Depreciation Method</th>
<th>5th Year Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Sinking Fund</td>
<td>570</td>
</tr>
<tr>
<td>Y</td>
<td>Straight Line</td>
<td>580</td>
</tr>
<tr>
<td>Z</td>
<td>Sum-of-the-Digits</td>
<td>630</td>
</tr>
</tbody>
</table>

The sinking fund rate is 5%. Which of the following is true?

(A) Z < Y < X  (B) X < Y < Z  (C) X < Z < Y  (D) Z < X < Y  (E) Y < Z < X

14. A machine is purchased for 50,000. The machine is expected to have a useful life of 15 years and a salvage value of 3000. The machine is depreciated using the Y% declining balance method, Y < 300. The depreciation charge in year 2 is 7219. Calculate the depreciation charge in year 15.

(A) 150  (B) 264  (C) 383  (D) 500  (E) 619

15. A 30-year 10,000 bond that pays 3% annual coupons matures at par. It is purchased to yield 5% for the first 15 years and 4% thereafter. Calculate the amount for accumulation of discount for year 8.

(A) 78  (B) 83  (C) 88  (D) 93  (E) 98

16. You are given two series of payments. Series A is a perpetuity with payments of 1 at the end of each of the first 2 years, 2 at the end of each of the next 2 years, 3 at the end of each of the next 2 years, and so on. Series B is a perpetuity with payments of \(K\) at the end of each of the first 3 years, 2\(K\) at the end of each of the next 3 years, 3\(K\) at the end of each of the next 3 years, and so on. The present values of the two series of payments are equal. Calculate \(K\).

(A) \(\frac{3i}{2}\)  (B) \(\frac{3d}{2}\)  (C) \(\frac{a_{3}}{a_{2}}\)  (D) \(\frac{a_{3}}{a_{2}}\)  (E) \(\frac{a_{3}}{a_{2}}\)

17. A loan of 10,000 is amortized by equal annual payments for 30 years at an effective annual interest rate of 5%. Determine the year in which the interest portion of the payment is most nearly equal to one-third of the payment.

(A) 6  (B) 7  (C) 8  (D) 23  (E) 25

18. Fund A is invested at an effective annual interest rate of 3%. Fund B is invested at an effective annual interest rate of 2.5%. At the end of 20 years, the total in the two funds is 10,000. At the end of 31 years, the amount in Fund A is twice the amount in Fund B. Calculate the total in the two funds at the end of 10 years.

(A) 5732  (B) 6602  (C) 7472  (D) 7569  (E) 812
19. You are given the following series of payments:

(i) \(100\) at time \(t = 1, 3, 5, \ldots, 19\)
(ii) \(200\) at time \(t = 2, 4, 6, \ldots, 20\).

An actuary is asked to determine the time, \(t^*\), such that the present value of the series of payments is equal to the present value of a single payment of 3000 made at time \(t^*\). Derive an exact expressions for \(t^*\), for \(i > 0\).

\[
\begin{align*}
(A) & \quad -\frac{1}{\delta} \cdot \log \left[ \frac{(1 + 2v)\bar{a}_{20}}{30\bar{a}_{21}} \right] \\
(B) & \quad -\frac{1}{\delta} \cdot \log \left[ \frac{(1 + 2v)\bar{a}_{20}}{30\bar{a}_{21}} \right] \\
(C) & \quad -\frac{1}{\delta} \cdot \log \left[ \frac{(v + 2v^2)\bar{a}_{20}}{30\bar{a}_{21}} \right] \\
(D) & \quad -\frac{1}{\delta} \cdot \log \left[ \frac{(v + 2v^2)\bar{a}_{20}}{30\bar{a}_{21}} \right] \\
(E) & \quad -\frac{1}{\delta} \cdot \log \left[ \frac{(v + 2v^2)\bar{a}_{20}}{30\bar{a}_{21}} \right]
\end{align*}
\]

20. The earnings of a corporation increase at 2% per quarter indefinitely. Each quarter the corporation plans to pay 40% of its earnings as a stock dividend. At the start of a quarter, an investor purchases the stock to yield a nominal rate of 10% compounded semi-annually. The first stock dividend is 2.00 payable at the end of the quarter. Calculate the theoretical price of the stock.

\[
\begin{align*}
(A) & \quad 406 \quad (B) 418 \quad (C) 426 \quad (D) 435 \quad (E) 447
\end{align*}
\]
PRACTICE MULTIPLE CHOICE TEST 5

1. You are given $\delta_t = \frac{2}{t-1}$ for $2 \leq t \leq 10$. For any one year interval between $n$ and $n+1$, with $2 \leq n \leq 9$, calculate the equivalent $d^{(2)}$.

   (A) $\frac{1}{n}$  
   (B) $\frac{2}{n}$  
   (C) $\frac{n-1}{n}$  
   (D) $\frac{n}{n-1}$  
   (E) $\left(\frac{n}{n-1}\right)^2$

2. The accumulated value of 1 at time $t$, for $0 \leq t \leq 1$, is given by a second degree polynomial in $t$. You are given (i) the nominal rate of interest convertible semiannually for the first half of the year is 5% per year, and (ii) the effective rate of interest for the year is 4% per year. Find the value of $\delta_{.75}$.

   (A) .021  
   (B) .023  
   (C) .025  
   (D) .027  
   (E) .029

3. Two funds, A and B, start with the same amount. Fund A grows at an annual interest rate of $i > 0$ for $n$ years, and at an annual interest rate of $j > 0$ for the next $n$ years. Fund B grows at an annual interest rate of $k > 0$ for $2n$ years. Fund A equals 1.5 times Fund B after $n$ years. The amounts in the two funds are equal after $2n$ years. Which of the following are true?

   I. $j < k < i$  
   II. $k < \frac{1}{2}(i+j)$  
   III. $j = k\left(\frac{2}{3}\right)^{\frac{1}{n}}$

   (A) I & II  
   (B) I & III  
   (C) II & III  
   (D) All  
   (E) None of A, B, C, D

4. You are given the following data on three series of payments.

<table>
<thead>
<tr>
<th>Payment at end of year</th>
<th>Accumulated value at end of year 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Series A</td>
<td>240</td>
</tr>
<tr>
<td>Series B</td>
<td>0</td>
</tr>
<tr>
<td>Series C</td>
<td>Y</td>
</tr>
</tbody>
</table>

Assume interest is compounded annually. Calculate $Y$.

   (A) 93  
   (B) 99  
   (C) 102  
   (D) 107  
   (E) 111

5. The force of interest at time $t$ is $\frac{e^{t}}{100}$. Calculate $a^{-1}(3)$.

   (A) .76  
   (B) .78  
   (C) .80  
   (D) .82  
   (E) .84

6. You are given an $n$-year annuity-due of 1 per year plus a final payment at time $n+k-1$, for $0 < k < 1$. The present value of the payments can be simplified to $\frac{1}{d} - \frac{1}{n+k}$. Calculate the final payment.

   (A) 1  
   (B) $k$  
   (C) $1 - k$  
   (D) $\frac{(1+i)^k - 1}{i}$  
   (E) $\frac{(1+i)^k - 1}{d}$
7. A corporation borrows 10,000 for 25 years, at an effective annual interest rate of 5%. A sinking fund is used to accumulate the principal by means of 25 annual deposits earning an effective annual interest rate of 4%. Calculate the sum of the net amount of interest paid in the 15th installment and the increment in the sinking fund for the ninth year.

(A) 664  (B) 674  (C) 684  (D) 694  (E) 704

8. The force of interest at time $t$ is $kt^3$. $R$ is the present value of a four-year continuously increasing annuity which has a rate of payment at time $t$ of $mt^3$. Calculate $R$.

(A) $\frac{k - me^{-4k}}{k}$  (B) $\frac{k - me^{-64k}}{k}$  (C) $\frac{1 - me^{-4k}}{k}$

(D) $\frac{1 - me^{-64k}}{k}$  (E) $\frac{m(1 - e^{-64k})}{k}$

9. Given that $i^{(4)} = .04$, calculate $(Da)_{68\mid}$.

(A) 1100  (B) 1109  (C) 1118  (D) 1127  (E) 1136

10. On a loan, payments of 1 are made at the end of each one-half of an interest conversion period for a total of five interest conversion periods. What is the amount of principal included in the eighth payment?

(A) $\frac{1}{2}v^{1.5}$  (B) $\frac{1}{2}v^{2.5}$  (C) $\frac{1}{2}v^{3.5}$  (D) $1 - \frac{1}{2}v^{1.5}$  (E) $v^{1.5}$

11. An annuity provides for 30 annual payments. The first payment of 100 is made immediately and the remaining payments increase by 8% per year. Interest is calculated at 13.4% per year. Calculate the present value of the annuity.

(A) 1423  (B) 1614  (C) 1753  (D) 1866  (E) 1944

12. John took out a 2,000,000 construction loan, disbursed to him in three installments. The first installment of 1,000,000 is disbursed immediately, and this is followed by two 500,000 installments at six month intervals. The interest on the loan is calculated at a rate of 15% convertible semiannually and accumulated to the end of the second year. At that time the loan and accumulated interest will be replaced by a 30-year mortgage at 12% convertible monthly. The amount of the monthly mortgage payment for the first five years will be one-half of the payment for the sixth and later years. The first monthly mortgage payment is due exactly two years after the initial disbursement of the construction loan. Calculate the amount of the 12th mortgage payment.

(A) 13,225  (B) 13,357  (C) 16,787  (D) 16,955  (E) 25,811
13. You are given the following 10-year bond with semi-annual coupons:

   (i) The purchase price is 650.
   (ii) The par value is 1000.
   (iii) The redemption value is 1050.
   (iv) The coupon rate is 12%.

   Using the bond salesman's method, calculate the nominal yield rate, convertible semiannually.

   (A) .0941     (B) .0952     (C) .1882     (D) .1904     (E) .1988

14. A common stock is purchased at a price equal to ten times current earnings. During the next eight years the stock pays no dividends, but earnings increase 50%. At the end of eight years the stock is sold at a price equal to 16.5 times earnings. Calculate the effective annual yield rate.

   (A) .052     (B) .065     (C) .083     (D) .102     (E) .120

15. A 700 par value five-year 10% bond with semiannual coupons is purchased for 670.60. The present value of the redemption value is 372.05. Calculate the redemption value.

   (A) 500     (B) 599     (C) 606     (D) 700     (E) 1000

16. A twenty-six week Treasury bill maturing for 10,000 is bought at a discount to yield 3.51% annually. For the same purchase price, a zero-coupon bond maturing for 50,000 at the end of 20 years is available. The nominal yield rate convertible semiannually on this bond is i. Calculate i.

   (A) 4.15%     (B) 4.25%     (C) 8.30%     (D) 8.50%     (E) 18.10%

17. A loan is to be repaid with five annual payments of P at an effective annual interest rate of i. The loan is paid off by the direct ratio method. A second loan is also to be repaid with five annual payments of P, but is to be amortized by the actuarial method at an effective annual interest rate of 5%. The balances outstanding at the end of the two years for the two loans are equal. Calculate $a_{5|1}$.

   (A) 4.21     (B) 4.24     (C) 4.28     (D) 4.31     (E) 4.41

18. A six-year loan of 100 is to be repaid by quarterly payments of 4.70 each. Rank the rates of interest estimated by each of the following:

   I. Constant ratio method
   II. Direct ratio method
   III. Merchant's rule

   (A) I < III < II     (B) II < I < III     (C) II < III < I     (D) III < I < II     (E) III < II < I
19. Simplify the following expression: \( \left( \frac{d}{dv} \delta \right) \cdot \left( \frac{dv}{d\delta} \right) \).

(A) \(-v^3\)  
(B) \(-v\)  
(C) 1  
(D) \(v\)  
(E) \(v^3\)

20. Plastic trays last 8 years and cost 20. Metal trays last 24 years and cost \(x\). Trays are needed for 48 years, and inflation will increase the cost of the trays 5% per year. At 10.25% interest, determine \(x\) so that the buyer is indifferent to purchasing plastic or metal trays.

(A) 36.90  
(B) 38.70  
(C) 40.70  
(D) 42.70  
(E) 44.80