1. A random sample $X_1, \ldots, X_n$ is taken from a normal distribution with an unknown mean $\mu$ and given variance $\sigma^2$.

   (a) Please explain how to derive a $(1 - \alpha)100\%$ confidence interval for $\mu$ using the fact that $\bar{X}$ is normal with mean $\mu$ and variance $\sigma^2/n$.

   (b) If $\sigma^2 = 12$, what is the smallest sample size for which the length of the 95% confidence interval for $\mu$ is less than or equal to 5?

2. Suppose that $X_1, \ldots, X_n$ is a random sample from a normal distribution with mean $\mu$ and variance $\sigma^2$. Assuming that the value of $\mu$ is unknown, explain how you would use the statistic $\sum (X_i - \bar{X})^2$ and a chi-square distribution to derive a $(1 - \alpha)100\%$ confidence interval for $\sigma^2$. Illustrate on a random sample of size $n = 100$ with $\mu = 10$, $\sigma^2 = 4$, and $\alpha = 0.05$ [You may want to use MINITAB - include a printout please].

3. Suppose $X_1, \ldots, X_n$ are i.i.d. random variables with an exponential distribution with mean $\theta$, that is with pdf: $f(x) = \frac{1}{\theta}e^{-x/\theta}$, $x > 0$. Show that $Y = \frac{2}{\theta}\sum_{i=1}^{n} X_i = \frac{2n}{\theta} \bar{X}$ has a $\chi^2$ distribution with $2n$ degrees of freedom.

4. Use the above result to derive a $(1 - \alpha) \times 100\%$ confidence interval for $\theta$. Illustrate by simulating a random sample of size 100 with mean equal to 10 (in MINITAB) and deriving a 95% confidence interval.

5. Suppose that $X_1, \ldots, X_n$ is a random sample from a normal distribution with mean $\mu_1$ and variance $\sigma_1^2$, and $Y_1, \ldots, Y_m$ is another random sample (independent of the first one) from a normal distribution with mean $\mu_2$ and variance $\sigma_2^2$.

   (a) Assuming that the variances are known, please derive a $(1 - \alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ using the fact that $\bar{X} - \bar{Y}$ has normal distribution.

   (b) Assuming that the two means are known, please derive a $(1 - \alpha)100\%$ confidence interval for $\sigma_2^2/\sigma_1^2$ using the fact that an appropriate statistic has an F distribution.