Correlations: C2, C3, C1

\[ \text{cov of } X_2 \text{ and } X_2 \] (unconcluded) Good!

\[ \text{cov of } X_1 \text{ and } Y \] high cov \( X_1 \text{ and } Y \) → Good!

\[ \text{cov of } X_2 \text{ and } Y \]

Regression Analysis: C1 versus C2, C3

The regression equation is

\[ C1 = 37.7 + 4.42 \times C2 + 4.37 \times C3 \]

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<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
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<th>T</th>
<th>P</th>
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<tbody>
<tr>
<td>Constant</td>
<td>37.650</td>
<td>2.996</td>
<td>12.57</td>
<td>0.000</td>
</tr>
<tr>
<td>C2</td>
<td>4.4250</td>
<td>0.3011</td>
<td>14.70</td>
<td>0.000</td>
</tr>
<tr>
<td>C3</td>
<td>4.3750</td>
<td>0.6733</td>
<td>6.50</td>
<td>0.000</td>
</tr>
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Estimated regression function

\[ b_1 = 4.42 \rightarrow \text{average increase in } Y \text{ for each 1 unit increase in } X_1 (\text{moisture content}) \]

when \( X_2 \) (sweetness) is held constant.

Box plot of residuals

Dish is symmetric → good!
 Res vs fits OK although there is some faint pattern here.

 Res vs X1 → OK

 Res vs X2 → OK

 Res vs X1 X2 → no pattern → good → interaction not really needed

 Normal plot of residuals → OK. No departure from normality.
Regression Analysis: C1 versus C2, C3

The regression equation is
C1 = 37.7 + 4.42 C2 + 4.37 C3

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SEE = 2.69330  R-Sq = 95.2%  R-Sq(adj) = 94.5%

Analysis of Variance

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<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>2</td>
<td>1872.70</td>
<td>936.35</td>
<td>129.08</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Residual Error</td>
<td>13</td>
<td>94.30</td>
<td>7.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>1967.00</td>
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(a) \[ H_0: \beta_1 = \beta_1 = 0 \quad \text{VS} \quad H_1: \beta_1 \text{ and } \beta_2 \text{ are not both equal to zero} \]

\[ F^* = \frac{MSR}{MSE} = \frac{936.35}{7.25} = 129.08 \]

Reject \( H_0 \) at \( \alpha = 0.01 \) level whenever \( F^* > F(1,13) \)

Since \( 129.08 > F = 6.70 \)

We reject \( H_0 \) and conclude \( H_1: \) Not both \( \beta_1 \) and \( \beta_2 \) are zero

(b) \[ P-value = P( \text{F-dash with 2 and 13 DF is} \geq 129.08) = 0.000 \quad \text{(see the output or check in MINITAB)} \]

(c) \( 1-\alpha = .99 \rightarrow \alpha = .01 \) For joint intervals for \( \beta_1 \) and \( \beta_2 \)

\[ b_1 \pm t \cdot s\{b_1\} \quad \text{and} \quad b_2 \pm t \cdot s\{b_2\} \]

Where \( t = t(1-\frac{.01}{2},13) = t(1-\frac{.005}{4},13) = t(.9975,13) = 3.37 \)

Also, \( b_1 = 4.42 \quad s\{b_1\} = 0.30 \quad b_2 = 4.375 \quad s\{b_2\} = 0.673 \) (see MINITAB output)

\( \beta_1: \ 3.41 - 5.43 \quad \beta_2: \ 2.1 - 6.64 \)
Regression Analysis: C1 versus C2, C3

Analysis of Variance

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The regression equation is

\[ C1 = 37.7 + 4.42 \times C2 + 4.37 \times C3 \]

Predictor | Coef  | SE Coef | T     | P     |
-----------|-------|---------|-------|-------|
Constant   | 37.650| 2.996   | 12.57 | 0.000 |
C2         | 4.4250| 0.3011  | 14.70 | 0.000 |
C3         | 4.3750| 0.6733  | 6.50  | 0.000 |

\[ S = 2.69330 \quad R^2 = \frac{SSR}{SSYD} = \frac{1872.70}{1967.00} \]

\[ R-Sq = 95.2\% \quad R-Sq(Adj) = 94.5\% \]

He written in $\hat{Y}$ is reduced by about 95% due to a linear relation with $X_1$ and $X_2$.

Predicted Values for New Observations

(a) New Obs | Fit | SE Fit | 99% CI | 99% PI |
-----------|-----|--------|--------|--------|
1          | 77.275 | 1.127 | (73.881, 80.669) | (68.481, 86.069) |

(b) Values of Predictors for New Observations

New Obs | C2 | C3 |
-------|----|----|
1      | 5.00 | 4.00 |

When $X_1 = 5$ & $X_2 = 4$

RS$\hat{S}(F)$

(a) Source | DF | SS    | MS     | F     | P     |
-----------|----|-------|--------|-------|-------|
Regression | 2  | 1872.70| 936.35 | 129.08| 0.000 |
Residual Error | 13 | 94.30  | 7.25   |       |       |
Total       | 15 | 1967.00|        |       |       |

(b) $H_0: \beta_2 = 0 \quad H_a: \beta_2 \neq 0$

\[ Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon \]

\[ F^* = \frac{SSR(X_2|X_1)}{1} = \frac{306.25}{7.25} = 42.24 \]

Since $F^* > F \rightarrow \text{REJECT } H_0 \rightarrow X_2 \text{ CAN NOT BE DROPPED!}$

\[ F(1, 13) = F(0.94, 1, 13) = 9.07 \]