1. The series converges by the AST since \( \frac{1}{\sqrt[3]{k}} \downarrow 0 \). Since \( \sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k}} \) is a \( p \)-series with \( p = 1/3 \) which diverges, the series \( \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{1/3}} \) is not absolutely convergent. It is conditionally convergent.

2. Since \( \frac{1}{k^2} \downarrow 0 \), the series converges by AST. Also, \( \sum_{k=1}^{\infty} \frac{1}{k^2} \) is convergent as a \( p \)-series with \( p = 2 \), hence the series \( \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \) is absolutely convergent.

6. Let \( a_k = b_k = \frac{(-1)^{k+1}}{\sqrt{k}} \). Then \( \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}} \) is convergent by AST, but \( \sum_{k=1}^{\infty} a_kb_k = \sum_{k=1}^{\infty} \frac{1}{k} \) is divergent.

7. Suppose one of the series \( \sum_{k=1}^{\infty} a_k^+ \) or \( \sum_{k=1}^{\infty} a_k^- \) is convergent. Since \( \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} a_k^+ + \sum_{k=1}^{n} a_k^- \) and \( \sum_{k=1}^{\infty} a_k \) is convergent, both series \( \sum_{k=1}^{\infty} a_k^+ \) and \( \sum_{k=1}^{\infty} a_k^- \) must be convergent. Since \( \sum_{k=1}^{\infty} |a_k| = \sum_{k=1}^{n} a_k^+ - \sum_{k=1}^{n} a_k^- \), it follows that \( \sum_{k=1}^{\infty} |a_k| \) is also convergent, contradiction.

11. Indeed, \( \sum_{k=0}^{\infty} 2^{-k} = \frac{1}{1 - 1/2} = 2 \). Let \( a_k = b_k = 2^{-k} \). Then

\[
\sum_{n=0}^{\infty} \sum_{k=0}^{n} a_kb_{n-k} = \left( \sum_{k=0}^{\infty} a_k \right) \left( \sum_{k=0}^{\infty} b_k \right) = 2 \cdot 2 = 4.
\]

But

\[
\sum_{n=0}^{\infty} \sum_{k=0}^{n} a_kb_{n-k} = \sum_{n=0}^{\infty} \sum_{k=0}^{n} 2^{-k} \cdot 2^{-n+k} = \sum_{n=0}^{\infty} \sum_{k=0}^{n} 2^{-n} = \sum_{n=0}^{\infty} (n+1)2^{-n}.
\]

We get that

\[
\sum_{n=0}^{\infty} (n+1)2^{-n} = 4.
\]
12. Consider \( a_k = b_k = \frac{(-1)^k}{\sqrt{k + 1}} \). The series \( \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{k + 1}} \) is convergent by the AST, but not absolutely convergent. We show that the series \( \sum_{n=0}^{\infty} \sum_{k=0}^{n} a_k b_{n-k} \) is divergent. Indeed,

\[
\sum_{k=0}^{n} a_k b_{n-k} = \sum_{k=0}^{n} \frac{(-1)^k(-1)^{n-k}}{\sqrt{k + 1} \sqrt{n - k + 1}} = (-1)^n \sum_{k=0}^{n} \frac{1}{\sqrt{(k + 1)(n - k + 1)}}.
\]

The maximum of the function \( f(x) = (x + 1)(n - x + 1) \) for \( 0 \leq x \leq n \) occurs at \( x = \frac{n}{2} \). It follows that

\[
\sum_{k=0}^{n} \frac{1}{\sqrt{(k + 1)(n - k + 1)}} \geq (n + 1) \frac{1}{\sqrt{\left(\frac{n}{2} + 1\right)^2}} = \frac{2(n + 1)}{n + 2} \geq 1.
\]

In particular \( \sum_{k=0}^{n} a_k b_{n-k} \) does not converge to 0, hence the series \( \sum_{n=0}^{\infty} \sum_{k=0}^{n} a_k b_{n-k} \) is divergent.