1. Determine if the series converges absolutely, converges conditionally, or diverges:

\[
\sum_{k=0}^{\infty} \frac{(-2)^k + 3^k}{4^k}, \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{k^3}, \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{4/2}}, \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{3+(-1)^k}},
\]
\[
\sum_{k=1}^{\infty} \frac{\cos(k\pi/3)}{k!}, \quad \sum_{k=1}^{\infty} \frac{(2k)!}{k!^2}, \quad \sum_{k=1}^{\infty} (\arctan k)^k, \quad \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} \arctan k.
\]

2. Find the interval of convergence for the power series

\[
\sum_{k=1}^{\infty} \frac{(-2)^k x^{2k}}{k^2}, \quad \sum_{k=2}^{\infty} \frac{(x + 3)^k}{2k \ln k}, \quad \sum_{k=2}^{\infty} \frac{(-1)^k}{k!(\ln k)^2}, \quad \sum_{k=1}^{\infty} \frac{x^k}{1 \cdot 3 \cdot 5 \cdots (2k - 1)}.
\]

3. Let \( f(x) = \sum_{k=0}^{\infty} c_k x^k \) where \( c_{2k} = 1 \) and \( c_{2k+1} = 2 \) for all \( k \geq 0 \). Find the largest interval on which \( f \) is continuous and an explicit formula for \( f(x) \) (not involving series).

4. Find the Taylor series around 0 for

\[
f(x) = \frac{x^2}{1 + x^2}, \quad \ell(x) = \ln \frac{1 + 2x}{1 - 2x}, \quad g(x) = \int \frac{\arctan x}{x} dx, \quad h(x) = \sin^2 x, \quad r(x) = \int \sqrt{1 + 4x^2} dx
\]
and determine their radius of convergence. Use the Taylor series to calculate \( f^{(10)}(0) \) and \( g^{(12)}(0) \).

5. Find the sum of the series

\[
a) \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k}}{k!}, \quad b) \sum_{k=0}^{\infty} \frac{(x + 2)^k}{(k + 2)!}, \quad c) \sum_{k=0}^{\infty} \frac{k + 1}{4^k}, \quad d) \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{3k^5}.
\]

6. Write the Taylor formula with \( a = 1 \) and \( R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x - 1)^{n+1} \) for

\[
a) \log(x + 2); \quad b) x^3 - x^2 + 5; \quad c) e^{2x}; \quad d) \frac{1}{x}.
\]
7. Let \( f(x) = \sum_{k=1}^{\infty} \frac{1}{(x+k)^2} \). Prove that \( f \) is continuous on \([0, \infty)\) and that \( \int_0^1 f(x) \, dx = 1 \).

8. By multiplying the geometric series with itself, show that for \(|x| < 1\) we have
\[
\frac{1}{2} \sum_{k=0}^{\infty} (k+1)(k+2)x^k = \frac{1}{(1-x)^3}.
\]

9. Let \( a \in \mathbb{R}^d \) be fixed, and let \( f : \mathbb{R}^d \to \mathbb{R}, f(x) = \|x - a\| \). Prove that \( f \) is uniformly continuous.

10. For the following sets determine their interior, closure and boundary:
\[
A = \{(x,y,z) \in \mathbb{R}^3 : 1 \leq \|(x,y,z)\| \leq 2\},
\]
\[
B = \{(x,y) \in \mathbb{R}^2 : \|(x,y)\|_1 < 1, y > 0\} \cup \{(0,-y) : y \geq 1\},
\]
\[
C = \{(x,y) \in \mathbb{R}^2 : 1 < \|(x,y)\|_\infty < 2\},
\]
\[
D = \{(x,y) \in \mathbb{R}^2 : y = \cos \frac{1}{x}, x \in (0,1]\} \cup \{(0,y) : y \in [-1,1] \cap \mathbb{Q}\}.
\]
Are they open, closed, bounded or compact?

11. Let \( K \subseteq \mathbb{R}^d \) be a compact set and let \( \{x_n\} \) be a sequence in \( K \). Prove that \( \{x_n\} \) has a convergent subsequence which converges to a point in \( K \).

12. On the set \( \mathbb{R} \) consider the metric \( \delta(x,y) = \begin{cases} 1 & x \neq y \\ 0 & x = y \end{cases} \). Describe the open balls and the closed balls with respect to this metric for \( r = 1/2, r = 1 \) and \( r = 2 \).

13. Consider a convergent sequence \( x_n \to x \) in \( \mathbb{R}^d \). Use the definition with open covers to prove that the set \( A = \{x_n : n \geq 1\} \cup \{x\} \) is compact.

14. Prove that an arbitrary intersection of compact sets in \( \mathbb{R}^d \) is compact.

15. If \( A, B \) are subsets of \( \mathbb{R}^d \), prove that \( (A \cap B)^c = A^c \cap B^c \). Show that in general \( (A \cup B)^c = A^c \cup B^c \) is false.

16. For \( E \subseteq \mathbb{R}^d \), prove that \( (E)^c = (E^c)^c \).

17. Find a subset \( A \) of \( \mathbb{R}^2 \) such that \( \partial A = \mathbb{R}^2 \).