1. Find the connected components of the complement $A^c$, where
   
   $$ A = \{(x, y) \in \mathbb{R}^2 : 1 < \|(x, y)\| \leq 2\} \cup \{(0, y) \in \mathbb{R}^2 : -2 < y < 2\}. $$

2. Prove that $S = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\} \setminus \{(0, 0, 0)\}$ is not connected. Find its connected components.

3. Give an example of a continuous function $f : \mathbb{R} \to \mathbb{R}$, and a connected set $L \subset \mathbb{R}$ such that $f^{-1}(L)$ is not connected.

4. Define $f(0, 0) = 0$ and $f(x, y) = \frac{x^3}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$.
   a) Prove that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are bounded on $\mathbb{R}^2$;
   b) Prove that $f$ is continuous on $\mathbb{R}^2$;
   c) Let $u$ be any unit vector in $\mathbb{R}^2$. Show that the directional derivative $D_u f(0, 0)$ exists and belongs to $[-1, 1]$;
   d) Prove that $f$ is not differentiable at $(0, 0)$.

5. Let $f(x, y) = 2x^3 - 3x^2 + 2y^3 + 3y^2$. Show that the gradient of $f$ vanishes at four points, and that $f$ has exactly one local maximum and one local minimum in $\mathbb{R}^2$.

6. Write the Taylor formula of degree 3 for $f(x, y) = \sin x \cos y$ at $(\pi, \pi)$.

7. For any $E \subset \mathbb{R}^n$, prove that the set of limit points of $E$ is closed. Give an example of a set in $\mathbb{R}$ with exactly four limit points.

8. Write an equation of the tangent 3-plane to the 3-surface $x^2 + y^2 - z^2 - w^2 = 4$ at $(2, 1, 0, 1)$.

9. Suppose $L : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear map such that $L(1, 1) = (a, b, a+b)$ and $L(-1, 2) = (b, a, b-a)$. Find the matrix of $L$.

10. If $A = \{(x, y) \in \mathbb{R}^2 : x^2 + 2y^2 \leq 1\}$ and $F : A \to \mathbb{R}^2$ is continuous, which of the following sets cannot be equal to $F(A)$? Justify your answer.
    a) $B_2(0, 0)$,  b) $\{(x, y) \in \mathbb{R}^2 : x^2 - y^2 \geq 1\}$,  c) $\overline{B}_1(0, 0) \cup \overline{B}_1(2, 2)$,  d) $\partial \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 2\}$.

11. Find the Taylor polynomial of degree 2 at $(1, 2, 1)$ for $h(x, y, z) = x^2y + z$. 


12. Find the extreme values of \( f(x, y) = x^3 - x + y^2 - 2y \) on the closed triangular region with vertices \((-1, 0), (1, 0), \) and \((0, 2)\).

13. Compute the limits

\[
a) \lim_{(x,y,z) \to (0,0,0)} \frac{xyz}{x^2 + y^2 + z^2}, \quad b) \lim_{x \to 1 \atop y \to 1} (x^2 + y^2 - 2x) \ln(xy - y), \\
c) \lim_{(x,y) \to (0,0)} \frac{\arctan(x^2 + y^2)}{xy}, \quad d) \lim_{(x,y) \to (0,0)} (x^2 + y^2) \sin \frac{1}{xy},
\]

or show that they do not exist.

15. Find the maximum value of \((xv - yu)^2\) subject to the constraints \(x^2 + y^2 = 1\) and \(u^2 + v^2 = 4\).

16. Let \(A \subset \mathbb{R}^d\) be bounded, and let \(f : A \to \mathbb{R}^d\) be uniformly continuous. Prove that \(f(A)\) is also bounded. Is this true for \(f\) just continuous?

17. Let \(B_1(0)\) be the open unit ball in \(\mathbb{R}^2\). Is it true that every continuous function \(f : B_1(0) \to \mathbb{R}\) takes Cauchy sequences into Cauchy sequences?

18. Suppose \(K \subset \mathbb{R}^d\) is compact, \(f : K \to \mathbb{R}\) is continuous such that \(f(x) > 0\) for every \(x \in K\). Prove that there is \(c > 0\) such that \(f(x) \geq c\) for every \(x \in K\).

19. Let \(F : \mathbb{R}^2 \to \mathbb{R}^3, F(x, y) = (\sin x \cos y, \sin x \sin y, \cos x)\). Find its differential matrix at \((\pi/4, \pi/2)\).

20. Let \(F : \mathbb{R}^3 \to \mathbb{R}, F(x, y, z) = x^3 y^2 z\). Determine its affine approximation at \((1, -1, 2)\).

21. Consider the series \(\sum_{k=0}^{\infty} (x^k, (1 - y)^k)\) in \(\mathbb{R}^2\). Determine the set where it converges pointwise and a set where it converges uniformly.

22. Let \(F : \mathbb{R}^3 \to \mathbb{R}\) be a \(C^1\) function and define \(G : \mathbb{R}^2 \to \mathbb{R}\) by

\[
G(x, y) = F(5x + 2y, xy, x^2 - 3y).
\]

Prove that \(G\) is \(C^1\) and determine \(dG(-1, 2)\) in terms of partial derivatives of \(F\).

23. Show that the sequence of functions \(\{\gamma_n\}_{n \geq 1}\) where \(\gamma_n : (0, 2] \to \mathbb{R}^2, \gamma_n(t) = \left(\frac{1}{1 + nt}, \frac{t}{n}\right)\) does not converge uniformly on \((0, 2]\).