1. Find the linearization \( L(x) \) of \( f(x) = \ln(1 + x) \) at \( a = 0 \) and use this to approximate \( \ln(1.01) \).

\[
\begin{align*}
  f(0) &= \ln 1 = 0 \\
  f'(x) &= \frac{1}{1 + x} \\
  f'(0) &= 1 \\
  L(x) &= 0 + 1(x-0) = x \\
  \ln(1.01) &\approx L(0.01) = 0.01
\end{align*}
\]
2. Express the limits as a derivative and evaluate

a) \( \lim_{x \to -1} \frac{x^4 - 1}{x + 1} \),

\[ b) \lim_{\theta \to \frac{\pi}{4}} \cot \theta - 1, \]

\[ c) \lim_{t \to \pi} \frac{e^{\sin t} - 1}{t - \pi}. \]

a) \( \lim_{x \to -1} \frac{x^4 - 1}{x + 1} = f'(-1) \) where \( f(x) = x^4 \), \( f'(x) = 4x^3 \)

\[ f'(-1) = -4 \]

b) \( \lim_{\theta \to \frac{\pi}{4}} \cot \theta - 1 = g'\left(\frac{\pi}{4}\right) \) where \( g(\theta) = \cot \theta \)

\[ g'\left(\frac{\pi}{4}\right) = -\csc^2 \theta \]

\[ g'\left(\frac{\pi}{4}\right) = -(\sqrt{2})^2 = -2 \]

c) \( \lim_{t \to \pi} \frac{e^{\sin t} - 1}{t - \pi} = h'(\pi) \) where \( h(t) = e^{\sin t} \)

\[ h'(\pi) = e \]

\[ h'(\pi) = e \cot \pi = -1 \]
3. Find $y'$ if

a) $\ln(xy) = x^2 - y^2$, b) $y = \sinh^{-1}(2x)$, c) $y = (\tan x)^{\arccos x}$.

\[ a) \quad \frac{y + xy'}{xy} = 2x - 2yy' \]
\[ \frac{1}{x} + \frac{y'}{y} = 2x - 2yy' \]
\[ y' = \frac{2x^2 - 1}{x} \cdot \frac{y}{2y^2 + 1} = \frac{2xy - y}{2xy^2 + x} \]

\[ b) \quad y' = \frac{2}{\sqrt{(2x)^2 + 1}} = \frac{2}{\sqrt{4x^2 + 1}} \]
\[ \ln(\tan x) \quad \arccos x \quad \arccos x \cdot \ln(\tan x) \]

\[ c) \quad y = e \]
\[ y' = e \cdot \arccos \ln(\tan x) \left( -\frac{1}{\sqrt{1-x^2}} \ln(\tan x) + \arccos x \cdot \sec^2 x \right) \]
\[ = (\tan x) \arccos x \left( \frac{\arccos x}{\sin x \cdot \cos x} - \frac{\ln(\tan x)}{\sqrt{1-x^2}} \right) \]
4. A window has the shape of a square surmounted by a right isosceles triangle with the hypothenuse identical to the side of the square. The base of the window is measured as having width 80 cm with a possible error in measurement of 0.2 cm. Use differentials to estimate the maximum possible error in computing the area of the window.

\[
AB = \frac{x}{\sqrt{2}} = \frac{\sqrt{2}x}{2} = \frac{x}{\sqrt{2}}
\]

\[
\text{Area (ABE)} = \frac{1}{2} \left( \frac{x}{\sqrt{2}} \right)^2 = \frac{x^2}{4}
\]

\[
\text{Area (BCDE)} = x^2
\]

\[
A(x) = x^2 + \frac{x^2}{4} = \frac{5}{4} x^2
\]

\[
dA = \frac{5}{4} 2x \, dx = \frac{5}{2} x \, dx
\]

\[
\Delta A \approx A'(x) \Delta x = \frac{5}{2} \cdot 80(0.2) = 40 \text{ cm}^2
\]
5. A paper cup has the shape of a cone with height 12 cm and radius 3 cm at the top. If water is poured into the cup at a rate of 3 cm³/s, how fast is the radius of the water level changing when the water is 4 cm deep? Recall \( V = \frac{1}{3} \pi r^2 h \).

\[
\frac{r}{h} = \frac{3}{12} = \frac{1}{4} \quad \Rightarrow \quad h = 4h
\]

\[
\sqrt(h) = \frac{1}{3} \pi h^2. \quad 4h = \frac{4\pi}{3} h^3
\]

\[
\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3h^2 \frac{dr}{dt} = 4\pi h^2 \frac{dr}{dt}
\]

When \( h = 4 \) \( r = 1 \)

\[
3 = 4\pi \cdot 1 \cdot \frac{dr}{dt} \quad \Rightarrow \quad \frac{dr}{dt} = \frac{3}{4\pi} \approx 0.2387 \text{ cm/s}
\]