1. a) Let \( g(x) = \int_{-1}^{\sin x} \ln(2 + t^2) dt \). Find \( g'(\pi) \).

b) Find \( f'(0) \) if \( f(x) = \int_{x^2}^{e^x} \sqrt{1 + t^2} dt \).

2. Use substitution to find

\[
\begin{align*}
a) \int \frac{\sin(\ln x)}{x} dx, & \quad b) \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx, & \quad c) \int_0^4 x \sqrt{16 - 2x} dx.
\end{align*}
\]

3. Use integration by parts to evaluate

\[
\begin{align*}
a) \int_0^{\pi/4} x \cos 2x dx, & \quad b) \int_0^1 x \ln(1 + x) dx, & \quad c) \int_1^\sqrt{3} x \arctan \frac{1}{x} dx.
\end{align*}
\]

4. A tank full of water has the shape of a hemisphere of radius 4 ft. Find the work required to pump the water out of the tank if water weighs 62.5 lb/ft³.

5. Evaluate the integrals

\[
\begin{align*}
a) \int_0^1 \sqrt{2x - x^2} dx, & \quad b) \int_1^2 x \sqrt{x^4 - 1} dx, & \quad c) \int_0^{\pi/2} \frac{\cos t}{\sqrt{1 + \sin^2 t}} dt, & \quad d) \int \frac{x}{\sqrt{1 + x^4}} dx.
\end{align*}
\]

6. Evaluate the trigonometric integrals

\[
\begin{align*}
a) \int_0^{\pi/2} \sin^5 x dx, & \quad b) \int \tan^3 x \sec x dx, & \quad c) \int_0^{\pi/4} \tan^4 t dt,
\end{align*}
\]

\[
\begin{align*}
d) \int \sin 8x \cos 3x dx, & \quad e) \int x \sec x \tan x dx, & \quad f) \int_\pi/6^{\pi/3} \csc^3 x dx.
\end{align*}
\]

7. Find the volume of the solid obtained when the region enclosed by \( x = 4y \) and \( y = \sqrt{x} \) in the first quadrant is rotated

\[
\begin{align*}
a) \text{about } x = 8; & \quad b) \text{about } y = 2.
\end{align*}
\]
8. Find the exact length of the curve given by
   
   \[ a) \quad y = 4(x - 1)^{3/2}, \quad 1 \leq x \leq 4; \quad b) \quad 12x = 4y^3 + \frac{3}{y}, \quad 1 \leq y \leq 3. \]

9. Find the volume of the solid when the following region is rotated about the y-axis:
   
   \[ a) \quad \text{under } y = \sin(x^2), \quad 0 \leq x \leq \sqrt{\pi}; \quad b) \quad \text{bounded by } x = 2y^2, \quad x = 1 + y^2. \]

10. First make a substitution to express the integrand as a rational function and then evaluate

   \[ a) \int \frac{dx}{x\sqrt{x - 1}}, \quad b) \int_0^1 \frac{1}{1 + \sqrt{x}} dx, \quad c) \int \frac{dx}{1 + e^x}, \quad d) \int \frac{\sin x}{\cos^2 x - \cos x} dx. \]

11. Evaluate the improper integrals if they are convergent:

   \[ a) \int_1^\infty \frac{\ln x}{x^2} dx, \quad b) \int_2^{10} \frac{y}{\sqrt{3y - 2}} dy, \quad c) \int_0^1 \frac{dx}{x^2 - 3x}. \]

12. Find the exact area of the surface obtained by rotating the curve about the x-axis:

   \[ a) \quad y = \sin \frac{x}{2}, \quad 0 \leq x \leq \pi; \quad b) \quad x = 1 + 2y^2, \quad 1 \leq y \leq 2. \]

13. A trough is filled with a liquid of density 840 kg/m³. The ends of the trough are equilateral triangles with sides 8 m long and with the vertex at the bottom. Find the hydrostatic force on one end of the trough.

14. Determine whether the series is conditionally convergent, absolutely convergent or divergent

   \[ a) \sum_{n=1}^{\infty} \frac{n}{n^3 + 1}, \quad b) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^n}, \quad c) \sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}, \quad d) \sum_{n=0}^{\infty} \frac{\cos n}{n^2 + 2}. \]

15. Verify that \( y_1 = t^{1/2} \) and \( y_2 = t^{-1} \) are solutions of \( 2t^2 y'' + 3ty' - y = 0. \)
16. Find the radius of convergence and the interval of convergence for the power series

\[ a) \sum_{n=0}^{\infty} \frac{(-3)^n x^n}{2n + 1}, \quad b) \sum_{n=1}^{\infty} \frac{(x - 1)^n}{2n(n + 3)^2}, \quad c) \sum_{n=0}^{\infty} \frac{3^n(x + 2)^n}{\sqrt{n + 3}}, \quad d) \sum_{n=0}^{\infty} \frac{n!x^n}{3^n}. \]

17. Find a power series representation for

\[ a) f(x) = \frac{x^2}{1 + x}, \quad b) g(x) = x \ln(2 - x), \]

\[ c) h(x) = \arctan \frac{1}{x}, \quad d) k(x) = x \sin 2x. \]

18. Find the Taylor series centered at \( a = 1 \) and the interval of convergence for

\[ a) f(x) = \ln x, \quad b) g(x) = \frac{1}{x}, \quad c) h(x) = \cos \pi x, \quad d) j(x) = e^{2x}. \]

19. Find the sum of the series

\[ a) \sum_{n=1}^{\infty} \frac{1}{n2^n}, \quad b) \sum_{n=2}^{\infty} \frac{1}{n^2 - n}, \quad c) \sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{n!}. \]

20. Solve the initial value problem \( 2(y - 1)y' = 3x^2 + 4x + 2, \ y(0) = -1. \)

21. A tank contains 100 L of pure water. Brine that contains 0.1 kg of salt per liter enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 8 minutes?

22. Use Euler’s method with step size 0.2 to estimate \( y(0.8) \) where \( y(x) \) is the solution of

\[ y' = 2xy^2, \quad y(0) = 1. \]

Find the exact solution and compare the value at 0.8 with the approximation.

23. Find the general solution of

\[ a) y' = 2 + 2x^2 + y + x^2y, \quad b) y' = 2x + y. \]