Entropy and optimality in river deltas

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Contributed by Andrea Rinaldo, August 29, 2017 (sent for review May 22, 2017; reviewed by Axel Kleidon and John B. Shaw)

The form and function of river deltas is intricately linked to the evolving structure of their channel networks, which controls how effectively deltas are nourished with sediments and nutrients. Understanding the coevolution of deltaic channels and their flux organization is crucial for guiding maintenance strategies of these highly stressed systems from a range of anthropogenic activities. To date, however, a unified theory explaining how deltas self-organize to distribute water and sediment up to the shoreline remains elusive. Here, we provide evidence for an optimality principle underlying the self-organized partition of fluxes in delta channel networks. By introducing a suitable nonlocal entropy rate (nER) and by analyzing field and simulated deltas, we suggest that delta networks achieve configurations that maximize the diversity of water and sediment flux delivery to the shoreline. We thus suggest that prograding deltas attain dynamically accessible optima of flux distributions on their channel network topologies, thus effectively decoupling evolutionary time scales of geomorphology and hydrology. When interpreted in terms of delta resilience, high nER configurations reflect an increased ability to withstand perturbations. However, the distributive mechanism responsible for both diversifying flux delivery to the shoreline and dampening possible perturbations might lead to catastrophic events when those perturbations exceed certain intensity thresholds.

Significance

River deltas are critically important Earthscapes at the land–water interface, supporting dense populations and diverse ecosystems while also providing disproportionately large food and energy resources. Deltas exhibit complex channel networks that dictate how water, sediment, and nutrients spread over the delta surface. By adapting concepts from information theory, we show that a range of field and numerically generated deltas obey an optimality principle that suggests that deltas self-organize to increase the diversity of sediment transport pathways across the delta channels to the shoreline. We suggest that optimal delta configurations are also more resilient because the same mechanism that diversifies the delivery of fluxes to the shoreline also enhances the dampening of possible perturbations.

Author contributions: A.T., A.L., D.A.E., I.Z., T.T.G., A.R., and E.F.-G. designed research, performed research, analyzed data, and wrote the paper.

Reviewers: A.K., Max Planck Institute for Biogeochemistry; and J.B.S., University of Arkansas.

The authors declare no conflict of interest.

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This article contains supporting information online at www.pnas.org/lookup/suppl/doi:10.1073/pnas.1708404114/-/DCSupplemental.

www.pnas.org/cgi/doi/10.1073/pnas.1708404114

PNAS Early Edition | 1 of 6
existence of an optimality principle in delta channel networks in terms of achieving configurations that maximize the diversity of flux delivery from any point of the network to the shoreline. The time scale of topologic reorganization in deltas, which is mainly through major channel avulsions, is of the order of decades to millennia, depending on the deltaic system (36, 37), whereas the flux adaptation on a given channel topology has a characteristic time scale which is orders of magnitude smaller. This separation of scales allows us to study the system on a fixed topology and focus on whether an optimality principle governs the distribution of fluxes on that topology.

Given the nature of our hypothesis, that is, maximization of the diversity (or uncertainty) of flux delivery from any point of the network to the shoreline, we adopt an information theoretic approach to quantify uncertainty based on Shannon entropy (38). Note that Jaynes (39) introduced a formalism demonstrating the equivalence (under thermodynamic equilibrium) of the statistical mechanics and information theory approach to entropy. Furthermore, it has been argued (not exempt from certain controversy) that the mathematical framework formulated by Jaynes (39) serves as a generalization of the statistical mechanics framework for both equilibrium and nonequilibrium systems (40, 41). In this work, we propose the notion of nonlocal entropy rate (nER) and suggest by comparative analysis of field and numerical deltas that indeed deltas self-adjust their flux partition so that they maximize their flux diversity. Note that by “maximization” we do not refer to a global optimization, that is, deltas generally do not achieve flux configurations that correspond to the absolute maximum value of nER but exhibit configurations corresponding to dynamically accessible (local) maximum (feasible optimality). We also discuss the possible implications of the proposed optimality principle with respect to delta resilience in response to perturbations, arguing that flux distributions characterized by extreme values of nER are more resilient, in that a local perturbation (e.g., flux reduction in a channel) will affect the least the distribution of fluxes at the shoreline outlets.

Nonlocal Entropy Rate (nER)

Entropy quantifies the uncertainty in the occurrence of events (38), that is, the amount of information needed to describe the outcome of an experiment. Uncertainty intuitively emerges from the notions of probability and surprise. For instance, given a discrete stochastic process \( \{X_i\} \), such as rolling a six-sided die, with specified probability distribution of outcomes, for example \( \{p_1, p_2, ..., p_n\} \), the occurrence of rolling a 3 when all sides are numbered 3 produces zero surprise. Conversely, on the same die rolling a number other than 3 would produce infinite surprise. Mathematically, this surprise is defined as \(-\log(p_i)\). Therefore, the uncertainty of an event, \( h_i \), is the product of \(-\log(p_i)\) times its probability of occurrence \( p_i \). Thus, either the occurrence of a completely certain event \( (p_i = 1) \) or an impossible one \( (p_i = 0) \) introduces zero uncertainty \( (h_i = 0) \), by convention \( 0\log 0 = 0 \). The uncertainty is maximal when all of the \( N \) possible outcomes have the same probability of occurrence \( 1/N \) (e.g., fair die where the number on each face has a probability of occurrence \( 1/6 \)). The total uncertainty or entropy, \( H \), of a set of \( N \) discrete outcomes with probabilities \( \{p_1, p_2, ..., p_N\} \) is equal to the sum of the uncertainties corresponding to each outcome \( i \) (42):

\[
H = \sum_{i=1}^{N} h_i = -\sum_{i=1}^{N} p_i \log p_i. \tag{1}
\]

We aim to develop an entropic metric for delta channel networks that quantifies the diversity of flux delivery to the shoreline. Thus, a delta channel network with low entropy, that is, with low uncertainty in flux pathways, would be one with a dominant channel that carries most of the flux. This configuration is inherently unstable, assuming no redistribution of sediment by marine processes, because that channel would prograde until its water slope reduces and the channel avulses down a steeper path (43, 44). However, a delta channel network that achieves a configuration where all of the possible pathways that drain water and

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**Fig. 1.** Field deltas and their corresponding location. Ten deltas with diverse morphodynamic environments and of various degrees of channel complexity were analyzed in terms of their nER (see SI Appendix for further details about the deltas). Satellite images provided by Landsat/Copernicus, NASA, Digital Globe, and CNES/Airbus were extracted from Google Earth. We acknowledge their respective copyrights.
sediment from any point of the network to the shoreline are equally probable would have the highest entropy. This uncertainty in delivery would have the stabilizing effect of spreading sediment evenly across the shoreline. Notice that maximizing the uncertainty of water and sediment flux delivery to the shoreline does not necessarily imply networks that proportion fluxes equally at every single bifurcation. In fact, this would be inconsistent with morphodynamic theories (45, 46) which showed that asymmetric flux partition at a junction is a requirement for stability.

Conceptualizing delta channel networks as graphs, where nodes correspond to junctions or bifurcations, and links represent channels (Materials and Methods), we define a metric of uncertainty of water and sediment flux pathways from the node $i$ to the shoreline as

$$h_{i}^{NL} = - \sum_{k} p_{ik} \log p_{ik},$$

where $p_{ik}$ is the transition probability from node $i$ to outlet node $k$. Alternatively, $p_{ik}$ represents the fraction of water and sediment flux from node $i$ that eventually drains to outlet $k$ (see Materials and Methods for details on the computation of $p_{ik}$). We refer to $h_{i}^{NL}$ as the nonlocal entropy for node $i$, to emphasize that the transition probabilities are between nodes $i$ and the shoreline nodes $k$ as opposed to among neighboring nodes (local). This notion acknowledges important nonlocal effects on delta dynamics, such as the hydrodynamic backwater where the water surface slope is dependent on the water depth at the shoreline and the local slope between subsequent bifurcations (36, 44).

By weighing $h_{i}^{NL}$ with the normalized steady-state flux at each node $i$, we define the $nER$ of the complete delta channel network:

$$nER = \sum_{i} \pi_{i} h_{i,d}^{NL} = \sum_{i} \pi_{i} \frac{\sum_{k} p_{ik} \log p_{ik}}{N_{i}},$$

where $h_{i,d}^{NL}$ is the normalized value of $h_{i}^{NL}$ and $N_{i}$ is the number of outlet nodes that can be reached from node $i$. The normalized $nER$ admits values in the interval $[0,1]$.

**Results and Discussion**

We hypothesize that deltas distribute the flux at each bifurcation to maximize the uncertainty in the delivery of fluxes from any point of the delta to the shoreline, that is, to achieve a dynamically accessible maximum of $nER$. We computed the $nER$ for 10 deltas with diverse morphodynamic environments and channel complexity (Fig. 1; see SI Appendix for the extracted channel networks and physical information about the deltas). We compared the computed $nER$ for each field delta (based on the actual flow partitions) against $10^5$ randomizations of the flux partition at each bifurcation [sampled from a uniform distribution in the interval $(0,1]$] holding the network structure constant. Despite the broad range of climate, discharge, and sediment influencing these deltas, all but one (the Niger delta) have flux configurations that exhibit extreme values of $nER$, that is, 9 of 10 field deltas have $nER$ above the 90th percentile of the random distribution (probability of exceedance $P_{E} < 0.1$) (Fig. 2). The fact that the computed value of $nER$ does not correspond to the absolute maximum of the distribution is not surprising. Natural systems have been argued to achieve stationary configurations that do not correspond to the absolute optimum of the functional describing their organizational principle but to local optima that are accessible given the initial conditions, constraints, and the system dynamics, known as the feasible optimality principle (19–22). Beyond field deltas, we also applied the $nER$ analysis to numerically simulated deltas that formed under varying incoming sediment grain sizes (32, 47, 48) using the physically based hydromorphodynamic model Delft3D (see SI Appendix for further details). The results show that five of the six numerical deltas exhibit extreme values of $nER$ with probability of exceedance $P_{E} < 0.1$ (Fig. 3), further supporting our optimality hypothesis. Note that the simulated delta ($D_{50} = 0.01$ mm) that does not satisfy the optimality of $nER$ is an extreme case, in terms of cohesiveness, for field deltas. Very cohesive banks are harder to erode and form levee breaches infrequently, delaying the triggering mechanism of avulsions. As a result, the system maintains itself at states at which the fluxes are not at equilibrium with its underlying channel network topology. This is reflected in suboptimal states of flux distribution and thus $nER$. 

![Fig. 2. $nER$ for 10 field deltas. Green stars represent the values of $nER$ computed for each field delta, using channel width (extracted from Landsat) as proxy for flux partition in bifurcations. We compared the values of $nER$ for each delta with $10^5$ randomizations of flux partitions (histograms). Nine out of the 10 deltas analyzed exhibit a maximal value of $nER$, defining maximal as a value where the probability of exceedance, $P_{E}$, by a random realization is less than 0.1.](image-url)
Our results suggest that the flux partitions at each bifurcation, which have evolved naturally, are not random but rather follow a rule that optimizes the delta system as a whole. In fact, an interesting paradox arises from our analysis. Although the entropy introduced locally by each bifurcation, considered as an insulated perturbation, is suboptimal (the maximum would correspond to a symmetric bifurcation which is not consistent with stability theory of bifurcation that dampens the intensity of perturbations and, finally, (v) we discussed the relation between entropy and resilience, arguing that delta flux configurations characterized by maximal \( nER \) are more resilient in the face of random perturbations. In the anthropocene where human activities have become a major agent of geomorphic change, understanding delta self-organization within an optimality perspective offers new ways of thinking about delta dynamics and disturbances that might hinder self-maintenance.

**Materials and Methods**

**Deltas as Directed Graphs.** Tejedor et al. (29, 30) presented a rigorous framework based on graph theory within which a delta channel network is...
represented by a directed graph, that is, a collection of vertices (bifurcations and junctions) and directed edges (channels in-between vertices, where the direction is given by the flow). All information about network connectivity and directionality of the flow can be stored in a sparse matrix called the adjacency matrix, $A$. Specifically, $A$ is an $N \times N$ matrix, where $N$ is the number of vertices, and whose entry $a_{ij}$ is unity if vertex $i$ receives fluxes directly from vertex $j$ (i.e., vertices $i$ and $j$ are connected by a link directed from $j$ to $i$) and zero otherwise. From $A$ we can derive an important matrix called Laplacian, which is equivalent to a diffusivity operator in a graph. To construct the Laplacian we need first to introduce the degree matrices for directed graphs. The in-degree (out-degree) matrix $D^{\text{in}} (D^{\text{out}})$ is an $N \times N$ diagonal matrix whose entries $d_i$ depict the number of links entering (exiting) vertex $i$ and are computed as the sum of the entries in the $i$-th row (column) of $A$. The Laplacian matrix, $L = (L^{\text{out}} - A)$, is defined as $D^{\text{in}} - A(D^{\text{out}} - A)$. Tejedor et al. (29, 30) showed that certain eigenvectors of the Laplacian operator contain important topological information of the deltaic network. Furthermore, more information about flux propagation can be obtained if $A$ is replaced with the weighted adjacency matrix $W$, where the weights $w_{ij}$ correspond to the fraction of flux in link $(i,j)$ with respect to the flux in vertex $i$. Similar to the at-station hydraulic geometry relationship (49)—width to landscape-forming discharge—reported for tributary rivers (50) and tidal channels (51), we assume the flux partition at the bifurcation to be proportional to the width of the downstream channels (18). Note that even though we do not consider explicitly in the computation of steady-state fluxes relevant processes such as water–sediment interchange between channel and islands (31, 52, 53), vegetation (54), tides (35), and so on, all of these processes set the hydrogeomorphometric attributes of the channel network. Therefore, the computation of steady-state fluxes in the channel network based on physical attributes such as channel widths can be interpreted to a certain degree as the result of the integrative effect of all of the main processes acting on a delta. For the purposes of this paper, there are two probability distributions that can be computed by simple algebraic manipulation in the above-mentioned operators, namely, the steady-state flux distribution and the node-to-outlet transition probability distribution.

**Steady-State Flux Distribution.** Having a delta represented as a directed acyclic graph (DAG) allows us to compute the steady-state flux by assuming conservation of mass. For instance, Tejedor et al. (29) showed how the steady-state flux can be formulated as an eigenvalue–eigenvector problem. For a DAG, there exists at least one indexing of the graph such that each offspring vertex has a higher index than its parent vertex. In this indexing, by construction, the matrix $W$ is strictly upper triangular, and therefore the matrix $W$ is nilpotent, guaranteeing the convergence of the sum as expressed in Eq. 4. In SI Appendix we prove that the stationary flux distribution $F$ can also be obtained as the stationary probability of a Markov process.

**Node-Outlet Transition Probability Distribution.** The node-to-outlet transition probability, $p_{ik}$, is defined as the probability that a package of flux at node $i$ drains to outlet $k$. Thus, the transport from each node $i$ to the different outlets $M$ can be understood as a discrete stochastic process with probability distribution $p_{ik}$, with $k = 1, \ldots, M$. Tejedor et al. (29) showed that with a delta channel network is represented by a directed acyclic graph $G$ with a weighted adjacency matrix $W$.

i) The null space of the weighted in-degree Laplacian $L^{\text{in}}(G^{\text{in}})$ for the reverse graph $G^{\text{out}}$ has dimension (multiplicity of the eigenvalue zero) equal to the number of outlets $M$.

ii) There exists a unique basis $\gamma_k$, $k = 1, \ldots, M$, of this null space in $\mathbb{R}^M$ (i.e., the basis consists of $M$ vectors each having $N$ entries) with the property

$$\gamma_k(i) = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases} \text{ for } k = 1, \ldots, M.$$
That is, the entry of the vector $\gamma_i$ corresponding to outlet $i$ is one, and zero at all other outlets.

thesis, if we define a matrix $T$, whose columns form the basis of the null space of $L_G^\prime\prime(G)$, $(\gamma_i)$, then the $i$-th row corresponds to the probability distribution $(\rho_k)$. 

Supporting Information (SI Appendix)
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Equivalent of the Input/output model to a Markov process
In this section, we prove the equivalence of the stationary probabilities that result from an irreducible and aperiodic discrete Markov process and the input/output model introduced in the Materials and Methods section.

Irreducible and aperiodic discrete Markov process. Let \( P \) be the transition probability matrix of a discrete Markov process which is irreducible (all the states are reachable from any state) and aperiodic (the return period to a given state can occur at different time steps),

\[
P = \{p_{ij}\}_{N \times N},
\]

where \( p_{ij} \) is the probability of transition from state \( j \) to state \( i \) at each time step. Then, we can define a stationary probability distribution \( \pi \):

\[
\pi P = \pi,
\]

where \( \pi = \{\pi_i\}_{N \times 1} \) is a column vector, whose entries correspond to the stationary probability distribution of each state \( i \). Therefore, \( \pi_i \) are non-negative values, satisfying \( \sum_{i=1}^{N} \pi_i = 1 \).

For a given directed acyclic graph, such as the ones we used to represent delta channel networks, if we assume conservation of mass, the dynamics of the system can be modeled by a Markov process where the outlets of the graph are reconnected to the apex with transition probability one. Thus, the transition probability matrix is equal to the weighted adjacency matrix of the graph, \( W \), if the entries \( w_{ij} \) corresponding to transition outlets to the apex are substituted by ones.

Equivalent of the Input/output model and Markov process solutions. We have shown in the Materials and Methods section that for a delta conceptualized as a directed acyclic graph fed from the most upstream node (apex) with a constant unit flux, we can compute the steady-state flux distribution \( F \) as:

\[
F = (I - W)^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.
\]

We prove in this section that the stationary distributions obtained from an irreducible and aperiodic Markov process, \( \pi \), and an input/output model, \( F \), are equivalent (up to a normalization factor). To prove this statement, we can decompose the transition probability matrix of the Markov process \( P \) as \( P = W + R \). \( R \) is called the recirculation matrix and it is defined as follows:

\[
R = \{r_{ij} = \delta_{i1}\delta_j(k)\}_{N \times N},
\]

where \( \delta \) represents the Kronecker delta; and therefore, all the entries of matrix \( R \) are zeros, except for the entries of the first row (apex has been indexed with \( i = 1 \) without loss of generality) that correspond to \( \{k\} \)-columns indexing the outlets.

Proof: If \( F = \pi \), then \( F \) must be an eigenvector of the probability transition matrix \( P \), and therefore \( PF = F \). Given,

\[
F = (I - W)^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}
\]

then,

\[
(W + R)(I - W)^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = (I - W)^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.
\]

By multiplying both sides of Eq. 6 from the left by \( (I - W) \),

\[
(I - W)(W + R)(I - W)^{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.
\]

Expanding the left side of Eq. 7,

\[
(I - W)W(I - W)^{-1} + (I - W)R(I - W)^{-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.
\]
Simplifying and rearranging Eq. 8,

\[
W\begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix}
+ (I - W)R(I - W)^{-1}
\begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix}
= \begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix},
\]

[9]

\[
(I - W)R(I - W)^{-1}
\begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix}
= (I - W)
\begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix},
\]

[10]

\[
R(I - W)^{-1}
\begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix}
= \begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix}.
\]

[11]

Defining \( B = R(I - W)^{-1} \),

\[
B
\begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix}
= \begin{pmatrix}
1 \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix}.
\]

[12]

Considering the structure of matrix \( R \) (see Eq. 4), then \( B \) is also a sparse matrix with the following entries:

\[
B = \{ b_{ij} \}_{N \times N}; b_{ij} = \sum_{l=1}^{N} r_{ii}(I - W)^{-1}_{ij} \delta_{i(l)} \delta_{l(k)},
\]

[13]

where the only non-zeros entries of the \( B \) matrix are in the first row (the apex has been indexed with \( l = 1 \) without loss of generality). Thus, the condition needed to satisfy Eq. 13 is simply that the entry \( b_{11} = 1 \). Therefore,

\[
b_{11} = \sum_{l=1}^{N} r_{ii}(I - W)^{-1}_{i1} \delta_{l(k)} = \sum_{l=1}^{N} (I - W)^{-1}_{i1} \delta_{l(k)} = 1. \]

[14]

In other words, the sum of the entries of the first column of \((I - W)^{-1}\) that corresponds to the outlets, \( \{k\} \), must be equal to one.

Given the definition of the input/output model (see Eq. 3), the first column of \((I - W)^{-1}\) stores the values of the fluxes, \( F \). Assuming conservation of mass, and that the input of the model is set to 1, the sum of the fluxes of the outlets must be one also, proving that the Markov model based on the idea of recirculating the flux and Input/Output model provide consistent stationary probability distributions.

Physical characteristics of the ten deltas analyzed

In this section, we summarize the physical characteristics of the ten deltas selected for analysis namely: Niger, Parana, Yukon, Irrrawaddy, Colville, Wax Lake, Mossy, Fraser, Danube and Mekong (Fig. 2). Extracting the channel networks from an air photo or satellite image of a delta is not an easy task. For this reason, we have adopted here for our preliminary analysis the exact five traced deltas in the study of Smart and Moruzzi (2) – Niger, Parana, Yukon, Irrrawaddy, and Colville – and have added the Wax Lake and Mossy deltas for which channel networks have been extracted in previous studies (3). We also added the channel networks of Fraser, Danube and Mekong extracted from Google Earth satellite images.

**Niger Delta**: The Niger delta is located in the West coast of Nigeria (latitude 4.95°, longitude 6.18°). It receives input from the Niger River with an average water discharge of 6,130 m^3 s^{-1} and sediment discharge of 3.97 x 10^7 tons yr^{-1} (4).
Niger delta is the largest delta in Africa covering an area of 24,508 km², sediment is mostly fine sand (5) and the tidal range is 3.0 m. The origin of the delta is estimated to be 80-35 million years BP during the Late Cretaceous (6). It is classified as a river dominated delta (4). By using the channel network extracted by Smart and Moruzzi (2), we identified 100 links, 71 vertices and 6 shoreline outlets.

Parana Delta: The Parana delta is located North of Buenos Aires, Argentina (-33.80°, -59.25°). It is fed by the Parana River, which delivers an average water discharge of 13,600 m³s⁻¹ and sediment discharge of 7.75 x 10⁷ tons yr⁻¹ (4). The Parana delta covers an area of 15,463 km² and sediment is mostly fine sand, silt and clay (8), and the tidal range is 4.0 m. Delta genesis was estimated during the Middle Holocene (6,000 years BP) (7). It is classified as a river and geology dominated delta (4). By using the channel network extracted by Smart and Moruzzi (2), we identified 140 links, 107 vertices and 20 shoreline outlets.

Yukon Delta: The Yukon Delta, located in the West coast of Alaska, USA (63.05°, -164.05°) receives input from the Yukon River with an average water discharge of 6,620 m³s⁻¹ and sediment discharge of 5.97 x 10⁷ tons yr⁻¹ (4). Parana delta covers an area of 15,463 km² and sediment is mostly fine sand, silt and clay (8), and the tidal range is 4.0 m. Delta genesis was estimated during the Middle Holocene (6,000 years BP) (7). It is classified as a wave dominated delta. We utilized the outline of the Yukon delta (3) containing 59 links, 56 vertices and 24 shoreline outlets.

Irrawaddy Delta: The Irrawaddy delta is located in the Southernmost coast of Myanmar (16.20°, 95.00°). It is fed by the Irrawaddy River at an average water discharge of 13,558 m³s⁻¹ and sediment discharge of 2.60 x 10⁸ tons yr⁻¹ (4). The delta covers an area of 6,438 km² with the deposited sediment composed of mostly mixed mud and silt (5), and the tidal range is 4.2 m. It is estimated that the delta began to form around 8,000-7,000 years BP together with most of the deltas in Southeast Asia (11). It is classified as a tide dominated delta (4). By using the channel network extracted by Smart and Moruzzi (2), we identified 181 links, 130 vertices and 15 shoreline outlets.

Colville Delta: The Colville delta, located in the Northern part of Alaska, USA (70.40°, -150.65°), receives input from the Colville River with an average water discharge of 491.7 m³s⁻¹ and sediment discharge of 1.16 x 10⁷ tons yr⁻¹ (12). With an area of 240 km², it is relatively small compared to other polar deltas. Sediment is mostly composed of gravel and sand (5). The tidal range is 0.2 m. The delta began to develop during the Middle Holocene (4,000 years BP) (13). It is classified as a river dominated delta (4). By using the channel network extracted by Smart and Moruzzi (2), we identified 140 links, 107 vertices and 20 shoreline outlets in the delta.

Wax Lake Delta: The Wax Lake delta, located in the coast of Louisiana, USA (29.51°, -91.44°), receives input from the Wax Lake outlet, a channel that was dredged in the early 1940s to mitigate flooding risk in the nearby Morgan City, at an average water discharge of 2,900 m³s⁻¹ and sediment discharge of 2.35 x 10⁷ tons yr⁻¹ (14). The slope of the Wax Lake delta from the delta apex to the Gulf of Mexico is 5.8 x 10⁻³ (15). Subaerial land only developed after the 1970s flood and has been experiencing rapid growth in the last two decades doubling to more than 100 km² today (16, 17). Sediment deposit in the delta is composed of approximately 67% sand (16), and the tidal range is 0.40 m (18). It is classified as a river dominated delta. We utilized the outline of the Wax Lake delta channel network processed by Edmonds et al. (3) containing 59 links, 56 vertices and 24 shoreline outlets.
Mossy Delta: The Mossy delta is located in Saskatchewan, Canada (54.07°, -102.35°). It is fed by the Mossy River with an average water discharge of 300 m³ s⁻¹ (3) and sediment discharge of 2.20 x 10⁶ tons yr⁻¹ (19). The delta was formed as a result of the avulsion of the Saskatchewan River in the 1870s (20). Progradation of the delta resulted in an area of 14 km² in the early 1940s (19) and after the construction of a spillway dam in the 1960s, the delta ever since slowly evolved with a current area of approximately 17 km². Sediment in the delta is roughly 50% fine-grained sand (3). Since the delta drains into a lake (Lake Cumberland), the effect of tides is insignificant. It is classified as a river dominated delta. We have extracted the channel network of Mossy delta and identified 67 links, 61 vertices and 23 shoreline outlets.

Danube Delta: The Danube delta is located in Romania (45.2°, 29.4°) and receives input from the Danube River with an average water discharge of 6,420 m³ s⁻¹ and sediment discharge of 6.72 x 10⁶ tons yr⁻¹ (4). It has an area of 6,468 km². Main control of the delta is waves (southern part) although the northern part is river-dominated (21). Recent studies show that the intensification of land use in the watershed as the population increased and land use technology has increased sedimentation in the delta (22).

Fraser Delta: The Fraser delta is located in Canada (49.18°, -122.95°) and receives input from the Fraser River with an average water discharge of 3,560 m³ s⁻¹ and sediment discharge of 2.00 x 10⁷ tons yr⁻¹ (4). It has an area of 876 km². Main control of the delta is river and tide. Recent studies show that the delta is experiencing more human intervention.

Mekong Delta: The Mekong delta is located in Vietnam (10.1°, 105.6°) and receives input from the Mekong River with an average water discharge of 14,770 m³ s⁻¹ and sediment discharge of 1.60 x 10⁷ tons yr⁻¹ (4). It has an area of 91,789 km². Main control of the delta is river and wave.

Delft3D Numerical Simulations

We use Delft3D to simulate the self-formed evolution of delta distributary networks. Delft3D is a physics-based morphodynamic model that has been validated for morphodynamics applications (23). We employ the depth-averaged version of Delft3D, which solves the unsteady shallow water equations in the horizontal dimension and assumes hydrostatic pressure in the vertical. Specifically, in this paper, we use model runs from Caldwell and Edmonds (24), which simulate a sediment-laden river entering a standing body of water that is devoid of waves, tides, and buoyancy forces. The river has an upstream water discharge boundary condition (steady flow of 1000 m³ s⁻¹) and carries sediment fluxes in equilibrium with the flow field. The downstream water surface boundary conditions are fixed at sea level. The flow field is coupled to the sediment transport equations (25, 26) and bed surface equations so it dynamically evolves in response to sediment transport gradients. The incoming sediment consists of grain sizes, D, lognormally distributed with a median size, $D_{50}$, and standard deviation $\sigma(\phi)$ (in φ space, where $\phi = \log_2 D$). We note that we use $D_{50}$ as the median of the incoming grain size distributions $D_{50}$, while the standard deviation is fixed to $\sigma(\phi) = 1$. The distributions have median sizes of 0.01 mm, 0.05 mm, 0.1 mm, 0.25 mm, 0.5 mm, and 1 mm, respectively (Fig. 3). These simulations are identical to runs B1a1, B1c1, B1e1, B1h1, B1m1, and B1o1 in Table 2 of Caldwell and Edmonds (24), exploring the whole range of cohesiveness (from 0% to 100%) and values of dominant grain size from 0.014 to 1.896 mm. For more discussion on the morphodynamics of these deltaic simulations, see Caldwell and Edmonds (24).

We utilized the capability of numerical simulations to investigate the change in nER during an avulsion cycle. Fig. 4 shows the avulsion cycle analyzed in this paper obtained from the run with $D_{50} = 0.10$ mm.

Channel Network Extraction and Analysis. The analysis conducted in this paper relies on spectral graph theory, which requires transforming each delta channel network into a graph. Graphs are mathematical objects composed of vertices and edges. For delta channel networks, the edges represent channels, and vertices correspond to the locations where one channel splits into new channels (bifurcation) or two or more channels merge into a single channel (junction). In pre-processing the gridded data produced by the simulations, we perform the following steps:

1. Classify pixels as Land/Channels/Ocean: First, we define a shoreline with the opening angle method (27) on a binary image where bed elevations below sea level were considered water and above sea level were considered land. We use an opening angle of 70°. All pixels not within
Fig. 4. Avulsion cycle. Five instances in an avulsion cycle of a Delft3D simulated delta where the main channel shifts, draining from the left (I) to the right (V) part of the delta shoreline. Intermediate stages of the avulsion cycle are also displayed: (II) the new path is created, (III) fluxes are equally divided between both paths, and (IV) the new path carries most of the flux. Top (bottom) panels are characterized by high (low) $nER$. Each panel is labelled with its corresponding time of simulation in hours (1 hr = 7.3 days morphodynamic time).

the shoreline are defined as ocean. Within the enclosed shoreline, pixels are defined as channels if depth > 0.25 m, velocity > 0.2 m s$^{-1}$, and sediment transport rate > 2.25 x 10$^5$ m$^3$s$^{-1}$. Everything else within the shoreline is defined as land.

2. Eliminate disconnected channels: From all the channel pixels, we only consider those ones that belong to channel pathways that eventually drain from the apex to the shoreline, removing isolated pixels and paths.

3. Extract skeleton network: We use an algorithm (28) to define the centerline of each channel, taking into account that channels can have a large range of variation in widths (from one pixel to several). From the resulting skeleton structure and flow directions, we define the vertices and edges that uniquely determine the directed graph corresponding to the delta channel network [e.g., see Tejedor et al. (1), Figure 7].

4. Compute adjacency matrix: All information about the network connectivity can be stored in a sparse matrix called adjacency matrix. The element of the matrix $a_{ij}$ is different from zero if the vertex $j$ is directly connected to downstream vertex $i$, and zero otherwise.

5. Extract channel widths: The width of channels measured directly downstream of each bifurcation is stored and used as a proxy for flux partition [see Tejedor et al. (1), section 2.2].


