EFFECTIVENESS OF LEAD-LAG PHASING ON PROGRESSION BANDWIDTH

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ABSTRACT

This paper provides quantitative assessments on two signal timing issues related to progression bandwidth maximization: 1) the effectiveness of using lead-lag phasing, and 2) the effect of number of signals on progression bandwidth. A computer program was developed to randomly generate multiple signal system scenarios and to provide maximal bandwidth solutions. The randomly generated signal system scenarios represented a wide variety of signal systems likely to be seen in the real world. Based on these randomly generated system scenarios and their associated maximal bandwidth solutions, conclusions were drawn regarding the two issues. It was found that lead-lag phasing had a significant advantage over the leading left-turn and lagging left-turn phasing schemes to provide maximal bandwidth solutions. At any signal in a system, lead-lag phasing was used in more than 70% of the cases compared to about 20% for leading left-turn phasing and 10% for lagging left-turn phasing. The number of signals had a profound impact on bandwidth attainability, showing a non-linear decline in attainability and bandwidth with an increasing number of signals in a system.

Keywords: Progression Bandwidth, Attainability, Signal System, Signal Phasing.

INTRODUCTION

Maximizing progression bandwidth is regarded as a primary objective when developing coordinated signal timing plans (1, 2). A larger progression bandwidth implies that more traffic on an arterial can progress through the signals without stops. Furthermore, bandwidth-based signal timing is preferred because it better meets drivers’ expectations. Besides general signal timing parameters of cycle length and phase split, phasing sequence is perhaps the most important element when maximizing progression bandwidth is desired (3, 4). Phasing sequence refers to the order of left-turn and through phases on the main street, which include dual leading left-turn (lead), dual lagging left-turn (lag), and lead-lag (see Figure 1). The effectiveness of using lead-lag phasing has been recognized by scholars and engineers for maximizing progression bandwidth. However, there has been no quantitative assessment on when lead-lag phasing can improve bandwidth over other phasing sequences, and how much bandwidth improvement can be achieved. Another related issue is how bandwidth is affected by the number of signals in a system.

The purpose of this study is to provide such quantitative evaluations. The primary objective of this study is, therefore, to answer the following two specific questions when seeking a maximal bandwidth solution for a signalized arterial: 1) how often is a lead-lag phasing needed at an intersection? 2) how will the maximal bandwidth change with an increasing number of signals in a system? To answer these two questions, a computer program was developed which can randomly generate hundreds of network cases.
and provide the maximal bandwidth solutions for each case. The cases were generated based on specific ranges of variable and parameter values, representing various signal system configurations likely to be seen in the real world. Based on the results from analyzing many such cases, sufficient data and valid statistics were gathered to draw definitive conclusions.

![Diagram of arterial street left-turn phasing sequences](image)

**Figure 1:** Typical arterial street left-turn phasing sequences

The remaining paper is organized as follows. A historical view of signal timing and related literature is given in the background section. Discussions on the development of the computer program are then provided, followed by analyses of the results from the computer program. Conclusions drawn from the study are presented at the end of the paper.

**BACKGROUND**

Pioneering research on signal timing optimization started with the development of mathematical models and associated computational algorithms. The first effort on this topic involved a computer program by Brooks (5) to maximize progression bandwidth given two-phase traffic signals along an arterial. Based on Brooks’ concept, Morgan and Little (6, 7) formulated a mathematical model for the bandwidth maximization problem. Their model used mixed-integer programming, from which an equal maximal bandwidth could be obtained for both directions. Since the publication of their work, many studies have followed in this area to improve modeling theories and techniques. The first major enhancement was the capability of handling multiphase traffic signals (8, 9, 10, 11) which led to the development of several popular computer software packages for signal timing optimization, including PASSER II (12), MAXBAND (9), and MULTIBAND (13, 14). Efforts to improve bandwidth-based optimization models
and their computing algorithms continued up to the late 1990’s, represented by the work of Papola (15, 16) and others (17, 18).

The work by Messer et al. (8) established the primary computing algorithm for the PASSER II and MAXBAND software packages. Maximizing progression bandwidth is achieved by minimizing a combined lower interference, $I_L$, and upper interference, $I_u$. The relationship between bandwidth and interference is illustrated in Figure 2 for a three-signal system of two-phase signals (i.e., no left-turn phases). In this figure, the one-direction bandwidth, $B$, is obtained by subtracting both the lower interference and the upper interference from the minimum arterial green, $G_{min}$.

![Figure 2: Bandwidth and interference](image)

For signals without left-turn phases, the lower and upper interferences for each intersection are determined by locating the signal offset at half-cycle increments, a technique that has been referred to as the half-integer optimization technique (8). For intersections with left-turn phases of equal length, the same half-integer optimization process can be applied. However, for intersections with left-turn phases of unequal phase length, the maximum bandwidth for both directions is obtained through a mixed-integer programming optimization process. Messer et al. (8) proposed an algorithm for maximizing the sum of progression bandwidth for both directions by first obtaining the maximal bandwidth for the outbound direction, and then minimizing the interference for the inbound direction. The outbound direction can be designated as the direction that has the higher minimum arterial green, i.e., $G_{o, min} \geq G_{i, min}$. The total maximum bandwidth for both directions, $B_{max}$, can be obtained by minimizing the inbound interference, $I_{i, min}$, using the following equations:

$$B_{max} = G_{o, min} + G_{i, min} - I_{i, min}$$  \hspace{1cm} (1)

$$I_{i, min} = \min\{\max(I_{U,j}) + \max(I_{L,j})\}; \quad j \in 1 - m; \quad j \neq x$$  \hspace{1cm} (2)
\[ I_{U,i,j,p} = G_{i,x} - (T_{x,j} + T_{j,x} - \gamma_{x,n} + \gamma_{j,p} + G_{i,j}) \]  
\[ I_{L,i,j,p} = T_{x,j} + T_{j,x} - \gamma_{x,n} + \gamma_{j,p} - G_{o,j} + G_{o,x} \]  
\[ 0 \leq I_{U,i,j,p} < C \]  
\[ 0 \leq I_{L,i,j,p} < C \]

Where

\[ B_{\text{max}} \] = the total maximum bandwidths for both directions (s),

\[ x \] = the intersection that has the minimum inbound green,

\[ m \] = number of signals in the system,

\[ I_{U,i,j,p}, I_{L,i,j,p} \] = upper and lower interferences caused by phasing sequence \( p \) of intersection \( j \),

\[ G_{i,x}, G_{o,x} \] = inbound and outbound green time at intersection \( x \), (i.e., \( G_{i,x} = G_{i,\text{min}} \))

\[ G_{i,j}, G_{o,j} \] = inbound and outbound green time at intersection \( j \),

\[ T_{x,j}, T_{j,x} \] = travel times between intersection \( x \) and intersection \( j \) in both directions,

\[ \gamma_{x,n} \] = the relative offset of \( G_{i,x} \) with respect to \( G_{o,x} \) for phase sequence \( n \),

\[ \gamma_{j,p} \] = the relative offset of \( G_{i,j} \) with respect to \( G_{o,j} \) for phase sequence \( p \), and

\[ C \] = cycle length.

From the above equations, it can be seen that interference increases (i.e., the bandwidth decreases) as the number of signals increases in a system. At a certain level, bandwidth may no longer be achievable (19). In assessing the effectiveness of a bandwidth solution, the measure of attainability is usually used. Attainability, \( A \), as defined in Equation 7, indicates how good a bandwidth solution is compared to the maximum possible solution for given traffic conditions and phase splits.

\[ A = (1 - \frac{I_{i,\text{min}}}{G_{o,\text{min}} + G_{i,\text{min}}}) \times 100 \]  

Theoretically, attainability has a minimum value for two-way arterials, which is:

\[ A_{\text{min}} = (1 - \frac{G_{i,\text{min}}}{G_{o,\text{min}} + G_{i,\text{min}}}) \times 100 = \frac{G_{o,\text{min}}}{G_{o,\text{min}} + G_{i,\text{min}}} \times 100 \]  

The minimum attainability implies that a one-direction progression bandwidth (the outbound direction) can always be obtained at \( G_{o,\text{min}} \); therefore, the minimum attainability should be no less than 50%. Attainability increases as the bandwidth for the inbound direction increases.
SOFTWARE DEVELOPMENT AND DATA ANALYSIS

A computer program was developed by the authors to derive maximal bandwidth signal timing solutions for multiple scenarios and assist in the data analysis. The program can randomly generate multiple traffic system scenarios based on specified ranges of key system elements, such as number of signals, travel times (to represent spacing and speed between signals), cycle length, and phase split. For example, each run of the program can automatically generate 100 different traffic system scenarios with different system characteristics. Based on each system scenario, the maximum bandwidth solution was derived according to the computing algorithm developed by Messer et al. (8), as described in the previous section. The phasing sequence at each intersection was also recorded. The program is currently limited to analyzing five signals. While the program can be extended to handle more intersections, the most important conclusions can still be drawn from the current analysis.

The following assumptions were made to generate the data and to conduct the analysis:

- Each intersection had exclusive protected left-turn phases on the arterial street, so that all the left-turn phasing sequences shown in Figure 1 can be used at all the intersections.
- No side-street phasing sequence was specified, because side-street phasing is irrelevant to arterial progression bandwidth.
- Cycle length ranged between 60 and 140 seconds.
- The total main-street phase split (effective green) was fixed at 60% of the cycle length, and the left-turn phase split ranged between 10% and 30% of the cycle length. The through phase split was automatically calculated once the left-turn phase split was determined, which varied between 30% and 50% of the cycle length.
- Travel time between adjacent intersections ranged between 15 and 25 seconds, representing different distance and travel speed. Different travel times in the two directions were allowed.

To analyze which phasing sequence is most effective for maximizing progression bandwidth, 100 signal system cases were randomly generated based on the above assumptions. Maximal bandwidth solutions were derived and the phasing sequences at each intersection used to produce the bandwidth solutions were recorded. It was noted that one particular system may have several maximum bandwidth solutions with different phasing sequences. In this case, all phasing sequences that produced the same maximal bandwidth solutions were recorded. Figure 3 through Figure 6 (representing analyses using 2 to 5 signals, respectively) show the results. In these figures, the frequencies in which a particular phasing had been used at each intersection are illustrated. Although 100 system scenarios are considered significant and the results should be representative to general cases in the field, variations from the true statistics still exist in
the results. In other words, slightly different results may occur if another 100 cases were to be generated (see Figure 5 and Figure 6).

**Figure 3:** Frequencies of each phasing sequence for the maximal bandwidth solutions – two signals

**Figure 4:** Frequencies of each phasing sequence for the maximal bandwidth solutions – three signals
Figure 5: Frequencies of each phasing sequence for the maximal bandwidth solutions – four signals

Figure 6: Frequencies of each phasing sequence for the maximal bandwidth solutions – five signals

Figure 3 (a two-signal system) shows that 84% of the cases involved using lead-lag phasing at both signals when the maximum bandwidth solutions were achieved. About 11% involved leading left-turn phasing and about 5% involve lagging left-turn phasing. It was concluded that in most cases, lead-lag phasing was more likely (at 84%) to produce the maximum bandwidth. Using leading left-turn phasing normally produced better bandwidth than using lagging left-turn phasing (11% vs. 5%). It was realized
that when the two left-turn phases had the same splits, leading left-turn and lagging left-turn would always produce exactly the same bandwidth, because both directions had the same start and end of the arterial green. However, when the two left-turn phases did not have the same splits, leading and lagging left-turn phasing produced different bandwidth solutions due to the different start of green for both directions. However, leading left-turn phasing usually produced better results than lagging left-turn phasing.

Figure 4 (a three-signal system) shows that the signal in the middle had a slightly higher frequency of using lead-lag phasing (78%) than the two signals at the ends (73%), i.e., the signal in the middle was more likely to use lead-lag phasing than the outer signals. Similarly, using leading left-turn phasing generally produced better bandwidth results than using lagging left-turn phasing. Similar trends can also be observed in Figure 5 (with four signals) and Figure 6 (with five signals).

Figure 7 shows the average frequencies of each phasing sequence for all signals when the maximum bandwidth solutions were achieved. As the number of signals in the system increased, the frequency of using lead-lag phasing decreased slightly, suggesting that some signals in the system could use other phasing sequences while still obtaining the maximum bandwidth solution. However, this decline seemed to level out as the number of signals exceeded five. Nevertheless, this frequency still remained high (over 70%), suggesting that lead-lag phasing was still the most effective for maximizing progression bandwidth.

Figure 7: Frequencies of using lead-lag phasing sequence for maximal bandwidth

Figure 8 shows how bandwidth attainability changed with the number of signals in a system. As can be seen, attainability decreased non-linearly with the increase of number of signals in the system. As discussed previously, attainability indicates how close the bandwidth solution is to its maximum possible
bandwidth. A 100% attainability indicates the absolute maximum bandwidth has been achieved. In theory, there is a minimum threshold for attainability as shown in the figure and expressed in Equation 8. When this threshold is reached, there will be no feasible bandwidth for the other direction. For cases analyzed, this threshold is projected to be met when the number of signals reaches seven. Therefore, when a signal system involves a large number of signals (e.g., more than seven signals), signal timing solutions just focusing on bandwidth may not be desired, and other signal timing techniques may be sought, such as system partition (19) or signal timing solutions focusing on partial progression (20, 21).

![Figure 8: Effect of number of signals on bandwidth attainability](image)

**SUMMARY AND CONCLUSIONS**

The paper addresses two issues related to bandwidth maximization for coordinated signal systems. A computer program was developed to randomly generate multiple signal system scenarios and to produce maximal bandwidth solutions. Based on the cases analyzed, the following major conclusions can be drawn:

- Lead-lag phasing showed significant advantages over other phasing sequences in maximizing progression bandwidth. For a signal system with five or fewer signals, over 70% of the cases used lead-lag phasing in order to produce a maximal bandwidth solution. This level seemed to remain constant with more signals in a system.

- In a system with more than two signals, signals in the middle were more likely to use lead-lag phasing than the signals at the ends.
• When the left-turn phases have the same phase lengths, leading left-turn phasing and lagging left-turn phasing did not show any difference in terms of maximizing progression bandwidth. However, when the two left-turn phases had different lengths, leading left-turn phasing showed some advantage over lagging left-turn phasing.

• Attainability tended to decline non-linearly with the increase of number of signals in a system. At a certain level, bandwidth may no longer be obtainable for one direction, suggesting that signal timing solely based on bandwidth may not be practical at that level.

The above conclusions were drawn based on analysis of systems with five or fewer signals. While the conclusions regarding the effectiveness of lead-lag phasing are not expected to change with more signals, further expansion of the program to analyze more than five signals is considered valuable to verify the effect of number of signals on bandwidth attainability.

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