Problem 1. For a function $u(x, y)$, its Laplacian is defined by

$$\Delta u = u_{xx} + u_{yy}.$$ 

Calculate $\Delta u$ for $u(x, y) = e^x \cos y$. Simplify as much as possible.

1. Failing to recognize that $\Delta u$, which is $e^x \cos y - e^x \cos y$, is equal to 0.
**Problem 2.** Find the moment of inertia about the origin for the lamina

\[ D = \{ x^2 + y^2 \leq R^2, \ x \geq 0 \} \]

with \( \rho(x, y) = \rho = \text{const.} \)

1. \( D = \{ 0 \leq \theta \leq \pi, \ 0 \leq r \leq R \} \). In fact, \( D = \{ -\pi/2 \leq \theta \leq \pi/2, 0 \leq r \leq R \} \). The former would be true if we had \( y \geq 0 \), not \( x \geq 0 \).

2. Not remembering the formula for \( I_0 \). Calculating center of mass or simply mass instead.

3. Calculating \( I_x \) and \( I_y \) separately and letting \( I_0 = I_x + I_y \). This is correct, but too long. Calculating \( I_0 \) is easier.
Problem 3. The same question for
\[ D = \{ -a \leq x \leq a, \ -a \leq y \leq a \}. \]

1. Calculating this for disc with radius \( R \). This is not a disc, this is a rectangle (a square, in fact).
2. Forgetting to multiply by \( \rho \).
3. Not remembering the formula for \( I_0 \). Calculating mass or center of mass instead.
Problem 4. Maximize $xyz$ under the conditions $x + y + z = 1$, $x, y, z \geq 0$. Find $x, y, z$ for which the maximum is achieved, and the value of this maximum. Do not forget the boundary!

1. Not considering the boundary:
   - $y = 0, 0 \leq x \leq 1$;
   - $x = 0, 0 \leq y \leq 1$;
   - $0 \leq x \leq 1, y = 1 - x$.
2. The line is $y = 1 - x$, not $y = x - 1$.
3. The system
   $f_x = x - x^2 - 2xy = 0, \quad f_y = y - y^2 - 2xy = 0$

   is solved in this way:
   
   $x(1 - x - 2y) = 0, \quad y(1 - y - 2x) = 0,$

   but we can cancel out $x$ and $y$, since we want to find only the points that lie strictly inside the domain. If you decide just to solve the system, without selecting the points strictly inside $D$, then writing just $(0, 0)$ and $(1/3, 1/3)$ is not sufficient, because $(1, 0)$ and $(0, 1)$ are also solutions.

4. Doing the Second Derivative Test and concluding that $(1/3, 1/3)$ is a maximum point. The Second Derivative Test implies only that this is a local maximum, it is not necessarily the maximum on the whole domain.
Problem 5. Find all positive $a$ such that the integral

$$\iint_D \frac{1}{\sqrt{x^2 + y^2}} dA$$

converges (i.e. its value is less than infinity), where

$$D = \{x^2 + y^2 \geq 1\}.$$

1. Writing $0 \leq r \leq 1$ instead of $1 \leq r \leq \infty$.
2. Saying that

$$\int_1^\infty \frac{dr}{r^{a-1}} = \log(r^{a-1})|_{r=\infty}^{r=1}.$$

In fact, for $a \neq 2$

$$\int_1^\infty \frac{dr}{r^{a-1}} = \frac{r^{2-a}}{2-a}|_{r=1}^{r=\infty}.$$

3. Saying: since

$$\int_1^\infty \frac{dr}{r^{a-1}} = \frac{1}{2-a},$$

then for $a = 2$ it is undefined. In fact, it is

$$\int_1^\infty \frac{dr}{r} = \log r|_{r=1}^{r=\infty} = \infty.$$

When $a = 2$, the integral also diverges.

4. After calculating

$$\int_1^\infty \frac{dr}{r^{a-1}} = \frac{1}{2-a},$$

you did not find: when does it actually converge, i.e. when its value is less than infinity?

$$\int_1^\infty \frac{dr}{r^{a-1}} = \frac{1}{2-a} \left( \lim_{r \to \infty} r^{2-a} - 1 \right).$$

If $2-a < 0$, this converges. If $2-a > 0$, this diverges. The case $a = 2$ is considered separately (see above).