Problem 1. [10 points] Suppose \( A = (1, 0, 0), \ B = (0, 1, 0), \) and \( C = (0, -2, 1). \) Find the angle \( A \) of the triangle \( ABC \) and its area. Round the angle up to the nearest degree.

The angle \( A \) (let us call it \( \theta \)) is the angle between the vectors \( \mathbf{a} = \overrightarrow{AB} = \langle -1, 1, 0 \rangle \) and \( \mathbf{b} = \overrightarrow{AC} = \langle -1, -2, 1 \rangle. \) So

\[
\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{(-1) \cdot (-1) + 1 \cdot (-2) + 0 \cdot 1}{\sqrt{(-1)^2 + 1^2 + 0^2} \sqrt{(-1)^2 + (-2)^2 + 1^2}} = \frac{-1}{\sqrt{2}\sqrt{6}} = -\frac{1}{2\sqrt{3}}.
\]

So the angle \( \theta \) is \( \arccos(-1/\sqrt{12}) \approx 107^\circ. \) The area \( S \) of this triangle is half the area of the parallelogram based on \( \mathbf{a} \) and \( \mathbf{b}, \) i.e.

\[
S = \frac{1}{2} |\mathbf{a} \times \mathbf{b}|.
\]

The cross product

\[
\mathbf{a} \times \mathbf{b} = \begin{vmatrix}
i & j & k \\
-1 & 1 & 0 \\
-1 & -2 & 1
\end{vmatrix} = \mathbf{i} + \mathbf{j} + 3\mathbf{k}.
\]

Thus,

\[
S = \frac{1}{2} \sqrt{1^2 + 1^2 + 3^2} = \frac{\sqrt{11}}{2}.
\]

Problem 2. [10 points] Find the plane passing through the line

\( x = 1 + t, \ y = 1 - t, \ z = 2t, \)

and the point \( P = (0, 4, 3). \)

Solution. We need to find a normal vector \( \mathbf{n} \) to this plane. It is orthogonal to this line, i.e. to its direction vector \( \mathbf{v}_1 = \langle 1, -1, 2 \rangle. \) Also, pick some point \( Q \) on this line, say \( Q = (1, 1, 0) \) (corresponding to \( t = 0 \)). Then \( \mathbf{n} \) is orthogonal to \( \mathbf{v}_2 := \overrightarrow{PQ} = \langle 1, -3, -3 \rangle. \) Therefore, we can take

\[
\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix}
i & j & k \\
1 & -1 & 2 \\
1 & -3 & -3
\end{vmatrix} = 9\mathbf{i} + 5\mathbf{j} - 2\mathbf{k} = \langle 9, 5, -2 \rangle.
\]

Thus, we have a normal vector to this plane and a point \( P = (0, 4, 3) \) on this plane, so we can write the equation of this plane:

\[
9(x - 0) + 5(y - 4) - 2(z - 3) = 0 \iff 9x + 5y - 2z = 14.
\]