Problem 1. [10 points] Find and classify critical points of
\[ f(x, y) = x^4 - 4x - y^2 + 2y. \]

Solution. \( f_x = 4x^3 - 4 \), \( f_y = -2y + 2 \). So critical points are the solutions of the system
\[ 4x^3 - 4 = 0, \quad -2y + 2 = 0 \iff x^3 = 1, \quad y = 1 \iff x = y = 1. \]
Therefore, \((1, 1)\) is the only critical point. Apply the Second Derivative Test:
\[ f_{xx} = 12x^2, \quad f_{xy} = 0, \quad f_{yy} = -2, \]
so at this point
\[ A = f_{xx} = 12, \quad B = f_{xy} = 0, \quad C = f_{yy} = -2, \quad D = AC - B^2 = -24 < 0, \]
and this is a saddle point.

Problem 2. [10 points] Using the linear approximation for \( f(x, y) = xy \) at \((x_0, y_0) = (1, 1)\), find the approximate value of \(1.01 \cdot 1.02\).

Solution. Since \( f(1, 1) = 1 \), and \( f_x = y \Rightarrow f_x(1, 1) = 1, \ f_y = x \Rightarrow f_y(1, 1) = 1, \) the linear approximation for \( f \) at \((1, 1)\) is
\[ f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) + f(1, 1) = (x - 1) + (y - 1) + 1 = x + y - 1. \]
If \( x = 1.01, \ y = 1.02, \) then \( x + y - 1 \approx 1.03. \) The exact value is, by the way, 1.0302. The difference is only 0.0002 = 0.02\%, very good!