Problem 1. [10 points] Approximate $1/1.1$ using $T_1(x)$ for $f(x) = 1/(1 + x)$ based at $b = 0$. Find the error bound for this approximation.

Solution. $f(0) = 1$, $f'(x) = -1/(1 + x)^2$, $f'(0) = -1$, so $T_1(x) = 1 - x$, and

$$\frac{1}{1.1} = f(0.1) \approx T_1(0.1) = 1 - 0.1 = 0.9.$$  

Since $f''(t) = 2/(1 + t)^3 \Rightarrow |f''(t)| = 2/(1 + t)^3$, the error bound is

$$\frac{0.1^2}{2} \max_{0 \leq t \leq 0.1} \frac{2}{(1 + t)^3} = \frac{10^{-2}}{2} \cdot \frac{2}{(1 + 0)^3} = 0.01.$$  

We used the fact that $2/(1 + t)^3$ is decreasing, i.e. the maximum over $0 \leq t \leq 0.1$ is obtained at the left endpoint: $t = 0$.

Problem 2. [10 points] Approximate $1/1.1$ using $T_3(x)$ for $f(x) = 1/(1 + x)$ based at $b = 0$. Find the error bound for this approximation.

Solution. $f(0) = 1$;  
$f'(x) = -1/(1 + x)^2 \Rightarrow f'(0) = -1$;  
$f''(x) = 2/(1 + x)^3 \Rightarrow f''(0) = 2$;  
$f'''(0) = -6/(1 + x)^4 \Rightarrow f'''(0) = -6$.

Thus,

$$T_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 = 1 - x + x^2 - x^3.$$  

And

$$\frac{1}{1.1} = f(0.1) \approx T_3(0.1) = 1 - 0.1 + 0.1^2 - 0.1^3 = 0.909.$$  

Since $f^{(4)}(t) = 24/(1 + t)^5$, and its absolute value is the same, it is a decreasing function and has its maximum over $0 \leq t \leq 0.1$ at $t = 0$, which is equal to 24. Thus, the error bound is

$$\frac{0.1^4}{4!} 24 = 10^{-4}.$$