Midterm one–Math 126 C/D, Winter 2011

Midterm one will be given on Wednesday, Jan. 26. It will cover Sections 10.1-10.3, 12.1-12.6 and 13.1-13.2.

Some basic rules

1. You may use a simple scientific calculator, but not graphing calculators.
2. You are allowed to have one page of hand-written notes of standard size.
3. Make sure to show all your work (except for True/False questions). You will not receive any partial credit unless all work is clearly shown.
4. Unless otherwise stated, always give your answers in exact form. For example, $3\pi$, $\sqrt{2}$, $\ln 2$ are in exact form, the corresponding approximations $9.424778$, $1.4142$, $0.693147$ are not in exact form.
5. There are five questions in the exam. Each question contains several parts.

Practice problems

Problem 1: True-False questions. Problems are similar to those in “Concept Check” and “True-False Quiz” on p. 669, pp. 812-813, and pp. 849-850.

Example: Determine if the following are True or False.

(a) For any vectors $\vec{u}$ and $\vec{v}$ in the 3-dimensional space, $\vec{u} \bullet \vec{v}$ is a vector in the 3-dimensional space.

(b) For any vector $\vec{u}$ and $\vec{v}$ in the 3-dimensional space, $\vec{u} \times \vec{v}$ is a vector in the 3-dimensional space.

(c) The cross product of two unit vectors is a unit vector.

(d) The line $\vec{r}(t) = (3-4t, 5-6t, -2+t)$ is parallel to the plane $-4x - 6y + z - 10 = 0$.

Problem 2: (Sections 10.1-10.3) Parametric curves in $\mathbb{R}^2$, tangent, area, arclength, polar curves, tangents to polar curves. Practice problems: p. 627, #24, 28, p. 648, #54, 55, 56 and p. 670, #3, 23, 25, 29, 33, 37, p672, #5.

Example: (a) The curve $C$ is defined by the parametric equations $x = 3t - t^3, y = 3t^2$ for $0 \leq t \leq 1$. Find the area between the curve $C$ and x-axis

(b) The curve $C$ is defined by the polar equation $r = \theta$ for $0 \leq \theta \leq \pi/2$. Find the arclength of $C$.

(c) Find the equation of the tangent line to the parametric curve $x = \sin^3 \theta, y = \cos^3 \theta$ at the point where $\theta = \pi/4$. 

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Problem 3: (Sections 12.1-12.4) Vector, length, angle, dot product, cross product, scalar projection, vector projection, triple product, direction angles, angle formulas, area and volume formulas. Practice problems: p785, #23, 25, 37, 39, p. 793, #31, 33, 37, p813, #3,5,7

Example: Let $\vec{u} = \langle 4, 3, 0 \rangle$ and $\vec{v} = \langle 5, 5, 5 \rangle$ and $\vec{w} = \langle 2, 3, c \rangle$. (i) Find a unit vector parallel to $\vec{u}$. (ii) Find the vector projection from $\vec{v}$ to $\vec{u}$. (iii) Find a vector orthogonal to both $\vec{u}$ and $\vec{v}$. (iv) Find the angle between $\vec{u}$ and $\vec{v}$. (v) Find the volume of the parallelepiped determined by the vectors $\vec{u}, \vec{v}, \vec{w}$. (vi) Find $c$ such that vectors $\vec{u}, \vec{v}, \vec{w}$ are coplanar. (vii) Find $c$ such that $\vec{u} \times \vec{v}$ is orthogonal to $\vec{w}$.


Example: (a) Consider the plane $x + y + z = 3$ and the line $L_1$: $\vec{r}(t) = \langle t - 1, 2t, -t + 1 \rangle$. (i) Determine whether the plane and the line intersect or parallel. (ii) If intersect, find the point of intersection. If parallel, find the distance between them. (iii) Find a line $L_2$ on the plane such that $L_1$ and $L_2$ are intersect and orthogonal.

Example: A surface consists of all points $P$ such that the distance from $P$ to $(1,0,-1)$ is twice the distance from $P$ to the plane $y = 1$. Find an equation for this surface and identify it.


Example: If $\vec{r}(t)$ is the position vector, then the velocity vector is defined to be $\vec{V}(t) := \vec{r}'(t)$ and acceleration vector is defined to be $\vec{a}(t) := \vec{r}''(t)$. Suppose acceleration vector is $\vec{a}(t) = \langle 3 \sin t, 3 \cos t, 0 \rangle$. (i) Find the velocity vector with an initial velocity $\vec{V}(0) = \langle -1, 0, 0 \rangle$. (ii) Find the position vector with an initial position $\vec{r}(0) = \langle 1, 1, 1 \rangle$.

Please also review double angle formula for $\sin x$ and $\cos x$. 

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