If a particle moves along a space curve defined by a vector function \( \mathbf{r}(t) \), then the velocity of the object is the vector \( \mathbf{v}(t) = \mathbf{r}'(t) \) and the acceleration is the vector \( \mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) \). The speed of the object is the magnitude of the velocity, i.e. \( |\mathbf{v}(t)| \).

**Example.** A particle starts moving from the origin with velocity \( \mathbf{v}_0 = <-1, -1, -1> \) with acceleration vector
\[
\mathbf{a}(t) = e^t \mathbf{i} + 3t^2 \mathbf{j} - 4 \sin t \mathbf{k}.
\]
Find velocity, speed, and position at time \( t \).

**Solution.**
\[
\mathbf{a}(t) = \langle e^t, 3t^2, -4 \sin t \rangle.
\]
Antidifferentiate to get \( \mathbf{v}(t) \):
\[
\mathbf{v}(t) = \langle e^t + C_1, t^3 + C_2, 4 \cos t + C_3 \rangle.
\]
But \( \langle 1 + C_1, C_2, 4 + C_3 \rangle = \mathbf{v}(0) = \mathbf{v}_0 = \langle -1, -1, -1 \rangle \), so we have:
\[
1 + C_1 = -1, \quad C_2 = -1, \quad 4 + C_3 = -1,
\]
therefore, \( C_1 = -2, \ C_2 = -1, \ C_3 = -5 \), and
\[
\mathbf{v}(t) = \langle e^t - 2, t^3 - 1, 4 \cos t - 5 \rangle.
\]
The speed is
\[
|\mathbf{v}(t)| = \sqrt{(e^t - 2)^2 + (t^3 - 1)^2 + (4 \cos t - 5)^2}.
\]
Then again antidifferentiate \( \mathbf{v}(t) \) to get \( \mathbf{r}(t) \):
\[
\mathbf{r}(t) = \langle e^t - 2t + K_1, \frac{t^4}{4} - t + K_2, 4 \sin t - 5t + K_3 \rangle.
\]
Since this particle starts moving from the origin, we have: \( \mathbf{r}(0) = \langle 0, 0, 0 \rangle \), so
\[
\langle 1 + K_1, K_2, K_3 \rangle = \langle 0, 0, 0 \rangle, \quad K_1 = -1, \ K_2 = K_3 = 0,
\]
and
\[
\mathbf{r}(t) = \langle e^t - 2t - 1, \frac{t^4}{4} - t, 4 \sin t - 5t \rangle.
\]