Quadric surfaces in $\mathbb{R}^3$ are counterparts of conic sections in $\mathbb{R}^2$. They are called \textit{quadric surfaces} because they involve first and second powers of variables. Their classification:

- Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
- Elliptic Paraboloid: $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
- Hyperbolic Paraboloid: $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$
- Cone: $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$
- Hyperboloid of One Sheet: $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- Hyperboloid of Two Sheets: $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

**Remark 1.** For the ellipsoid, the $x$, $y$ and $z$ intercepts (intersections with $x$, $y$ and $z$ axes) are $\pm a, \pm b$ and $\pm c$ respectively. (Just plug in, e.g. $y = z = 0$ to find $x$ intercepts.) This ellipsoid is centered at the origin. If $a = b = c$, this is a sphere.

**Remark 2.** For the elliptic paraboloid, the variable raised to the first power indicates the axis of the paraboloid.

**Remark 3.** How to distinguish between the two hyperboloids? Take the two variables which have the same sign. Here it is $x$ and $y$. Plug in $x = y = 0$. If you got an equation without solution (here it is $z^2/c^2 = -1$), this is a hyperboloid of two sheets, since it does not intersect with the $xy$-plane; the first sheet is above this plane, the second one is below. If your equation has solution (here it is $z^2/c^2 = 1$), this is a hyperboloid of one sheet, since it does intersect with the $xy$-plane.

The axis in both cases is indicated by the variable with positive coefficient.

**Remark 4.** If you change the roles of $x, y, z$, or add some constants to them, the type of surface will remain the same! You know this from two dimensions: $y = x^2$ is a parabola, but $(y - 1)^2 = 2x$ is also a parabola. For example, the surface

$$y^2 + (z - 2)^2 - 2(x + 1)^2 = 2$$

is a hyperboloid of two sheets. Indeed,

- after a shift, it takes the form $y^2 + z^2 - 2x^2 = 2$;
- divide it by $-2$:
  $$x^2 - \frac{z^2}{2} - \frac{y^2}{2} = -1;$$
- change the roles of $x, y, z$:
  $$z^2 - \frac{x^2}{2} - \frac{y^2}{2} = -1.$$  

Here, $c = 1$, $a = b = \sqrt{2}$.

**Remark 5.** If any of the coordinates is not present in the equation, then this is a cylinder. For example, $y = x^2$ is a parabolic cylinder in $\mathbb{R}^3$; $z^2 - x^2 = -1$ is a hyperbolic cylinder in $\mathbb{R}^3$. 