Consider the equation
\[ y'' + 8y' + 16y = e^{2t} + 1 + te^{-4t}. \]

The general solution \( y \) is equal to a particular solution \( y_1 \) of the nonhomogeneous (original) equation plus the general solution \( y_0 \) of the homogeneous equation. First, solve the corresponding homogeneous equation:
\[ y'' + 8y' + 16y = 0. \]

Characteristic equation: \( \lambda^2 + 8\lambda + 16 = 0 \), double root \(-4\). Therefore, the general solution of the homogeneous equation is
\[ y_0(t) = C_1e^{-4t} + C_2te^{-4t}. \]

Now,
\[ y_1 = y_2 + y_3 + y_4, \text{ where } \]
\[ y''_2 + 8y'_2 + 16y_2 = e^{2t}, \]
\[ y''_3 + 8y'_3 + 16y_3 = 1, \]
\[ y''_4 + 8y'_4 + 16y_4 = te^{-4t}. \]

Now, \( e^{2t} \) is a polynomial of degree zero (namely, 1) times an exponent, but \( 2 \neq -4 \). Therefore, we do not need to raise the degree of the polynomial: try to find \( y_1 = A_1e^{2t} \). Then
\[ y'_2 = 2A_1e^{2t}, \quad y''_2 = 4A_1e^{2t}, \]
and
\[ y''_2 + 8y'_2 + 16y_2 = 36A_1e^{2t} = e^{2t} \Rightarrow A_1 = \frac{1}{36}. \]

So \( y_2 = \frac{1}{36}e^{2t} \). Now, for \( y_3: 1 = 1 \cdot e^{0t}, \) and so this is again a polynomial of degree zero times an exponent with \( 0 \neq -4 \). So we try \( y_3 = A_2 \). Then \( y'_3 = y''_3 = 0, \) and
\[ y''_3 + 8y'_3 + 16y_3 = 1 \Rightarrow 16A_2 = 1 \Rightarrow A_2 = \frac{1}{16}. \]

Finally, \( y_4: \) since the right-hand side is a polynomial of degree 1 times \( e^{-4t} \), then we raise this degree by 2. We get:
\[ y_4 = (A_3t^3 + A_4t^2) e^{-4t}. \]

The numbers \( A_3, A_4 \) are to be found. We do not need to write the terms \( (A_5t + A_6) e^{-4t} \), since they are already included into \( y_0 \) (and anyway we will not be able to determine \( A_5, A_6 \), since these are arbitrary constants \( C_1, C_2 \)). Therefore, the answer is
\[ y = y_0 + y_1, \quad y_1 = y_2 + y_3 + y_4, \]
\[ y_0 = C_1te^{-4t} + C_2e^{-4t}, \quad y_2 = \frac{1}{36}e^{2t}, \quad y_3 = \frac{1}{16}, \quad y_4 = (A_3t^3 + A_4t^2) e^{-4t}. \]

**Example.** \( y'' = t^2 + te^t \). We can solve it using two integrations:
\[ y' = \frac{t^3}{3} + \int te^t = \frac{t^3}{3} + te^t - e^t + C_1, \]
\[ y'' = \frac{t^4}{12} + te^t - e^t - e^t + C_1t + C_2. \]
But we can also use the general theory: $\lambda^2 = 0$, and $\lambda = 0$ is the double root, so the solution to the homogeneous equation $y'' = 0$ is $y = C_1 t e^{0t} + C_2 e^{0t} = C_1 t + C_2$. The part $t^2 = t^2 e^{0t}$ gives us $y = At^4 + Bt^3 + Ct^2$, and the part $te^t$ gives us $(Dt + E)e^t$. So the general solution:

$$C_1 t + C_2 + At^4 + Bt^3 + Ct^2 + (Dt + E)e^t$$

has the same form as above.