
Assume you borrowed a college loan of 50000$. You got a good job after you graduated and you are able to pay back 20000$ every year. The annual interest rate is 10%. But interest is accrued continuously, and you pay back also continuously.

Assume $y(t)$ is the amount you owe at time $t$. For simplicity, let $1 = 10000$ $\$ \ $\text{(to avoid writing zeros all the time).}$ Take a small $dt$. During a time interval $[t, t+dt]$, the interest accrued is $y(t) \cdot 10\% dt = y(t) \cdot 0.1 dt$. Indeed, the annual interest rate is 10%, and so the interest rate for the part $dt$ of a year is 10$\% dt$. Since during this small time interval the amount $y(t)$ of money you owe does not change, the interest accrued is equal to the interest rate for this time interval times the amount of debt, that is, $y(t) \cdot 10\% dt = y(t) \cdot 0.1 dt$.

And you pay $2 dt$ during this time interval. Indeed, you pay 2 every year, continuously, and so you pay $2 dt$ during time $dt$.

Debt at time $t + dt = \text{debt at time } t + \text{interest accrued - payments.}$ So

$$y(t + dt) = y(t) - 2 dt + y(t) \cdot 0.1 dt.$$  

Rewrite this as

$$\frac{y(t + dt) - y(t)}{dt} = -2 + 0.1 y(t) \Rightarrow y'(t) = \frac{y(t)}{10} - 2.$$

The initial condition is $y(0) = 5$ (because the initial debt is 50000$ = 5$). Let us solve this initial value problem

$$y' = \frac{y}{10} - 2, \quad y(0) = 5.$$

This is a linear nonhomogeneous equation, and it is also separable. You may solve it either using variation of parameters (as a linear equation), or separation of variables (as a separable equation). Let us use variation of parameters. The corresponding linear homogeneous equation:

$$y' = \frac{y}{10} \Rightarrow y = Ce^{t/10}.$$

Variation of parameters: let $C = C(t)$ and plug $y = C(t)e^{t/10}$ into the original nonhomogeneous equation $y' = y/10 - 2$. We get:

$$y' = C'(t)e^{t/10} + \frac{C(t)}{10} e^{t/10} = \frac{y}{10} - 2 = \frac{C(t)}{10} e^{t/10} - 2.$$

Therefore, 

$$C'(t)e^{t/10} = -2 \Rightarrow C'(t) = -2e^{-t/10}.$$

Integrate this: using 

$$\int e^{at}dt = \frac{1}{a}e^{at} + K;$$

we let $a = -1/10$ and get:

$$C(t) = \int (-2)e^{-t/10} = (-2)(-10)e^{-t/10} + K = 20e^{-t/10} + K.$$

Plug in back into $y(t) = C(t)e^{t/10}$:

$$y(t) = (20e^{-t/10} + K)e^{t/10} = 10 + Ke^{t/10}.$$
This is a general solution to the equation $y' = y/10 - 2$. Now, let us find the value of $K$ which corresponds to the initial value problem.

$$y(0) = 20 + K = 5 \Rightarrow K = -15.$$ 

Therefore, we have:

$$y(t) = 20 - 15e^{t/10}$$

This is the amount of money you owe at time $t$. When will you get rid of the debt? Let us find $t$ such that $y(t) = 0$. This is

$$20 = 15e^{t/10} \Rightarrow e^{t/10} = 4/3 \Rightarrow t = 10 \log(4/3) \approx 2.88$$

If there were no interest, then you would pay the loan back in $50000/20000 = 2.5$ years. In the presence of the interest rate, you will pay it back in 2.88 years.