Homework 1, due October 8, 2014  
Math 307J. Autumn 2014

For each of the differential equations 1-10, find the general solution.

**Problem 1.** $y' = \lambda y$, $\lambda \in \mathbb{R}$ is a real constant.

**Problem 2.** $y' = -2y + 1$.

**Problem 3.** $y' = t^2y^2$.

**Problem 4.** $y' = y^3e^t$.

**Problem 5.** $y' = e^{t+y}$.

**Problem 6.** $y' = (y^2 + 1)/(t^2 + 1)$.

**Problem 7.** $y' = 2y - t - 1$.

**Problem 8.** $y' = y/t + t^2$.

**Problem 9.** $y' = y(1 - y)$ (logistic growth).

**Problem 10.** $ty' + 3y = t^3$.

**Problem 11.** Solve the initial value problem $y(0) = 2$ for the equation from Problem 3.

**Problem 12.** Solve the initial value problem $y(0) = 1$ for the equation from Problem 4.

**Problem 13.** Solve the initial value problem $y(1) = 0$ for the equation from Problem 8.

**Problem 14.** Solve the initial value problem $y(-1) = 0$ for the equation from Problem 9.

**Problem 15.** Consider a pond with volume 1000. It contains volume 1 of toxic waste. The waste is dissolved in the pond, so the result is the well-stirred solution. We would like to clean this pond, so with rate 2 per minute the solution flows out of the pond. With rate 1 per minute, the clean water flows in the pond. What is the concentration of the toxic waste in 100 minutes?

**Problem 16.** Suppose that you have 1000 in a bank. The annual interest rate is $r = 2\%$, accrued continuously. (i) What amount will you have after one year? (ii) How long do you need to wait until your savings double (that is, you have 2000)?

**Problem 17.** A home buyer can afford to spend 1000 on mortgage payments per month. He pays continuously, and the annual interest rate us 5%. The interest is also accrued continuously. Determine the amount he can borrow for 15-year mortgage.

**Problem 18.** A ball with mass $m$ is thrown into the air with initial velocity $v_0$. The gravitational acceleration is $g$. Find the maximum height which the ball reaches. When does it reach this maximum height?

**Problem 19.** A ball with mass $m$ is thrown into the air with initial velocity $v_0$. This time, it encounters air resistance force which is proportional to $v$: $F = kv$. What is the maximal height it reaches? (Hint: at the moment of the maximal height, $v = 0$.)