In Problems 1-4, write the general solution of the equation with undetermined coefficients (but do not find the coefficients). Example:

\[ y'' - y = 2te^t \Rightarrow y = C_1 e^t + C_2 e^{-t} + (At^2 + Bt)e^t. \]

Indicate which constants are arbitrary parameters (here: \( C_1, C_2 \)) and which are concrete numbers, determined by plugging into the equation (here: \( A, B \)).

**Problem 1.**

\[ 4y'' + 4y' + y = \frac{1}{2}te^{-t} + 2 - e^{-t/2}. \]

**Problem 2.**

\[ y'' + 4y' + 8y = (-t^2 + 3)e^{-2t} \cos(2t) + e^{-2t} + 3t^3. \]

**Problem 3.**

\[ y'' - 4y' + 3y = e^{4t} + (2 - t)e^{3t}. \]

**Problem 4.**

\[ y'' - 2y' + y = (t^3 + 2t)e^t + t^2 + (t^2 + 34)e^{3t} \sin(2t). \]

**Problem 5.** Determine the steady-state solution for mechanical vibrations with \( m = 1, k = 2, \gamma = 1, F_0(t) = 3 \cos t \).

**Problem 6.** For \( F_0(t) = -2 \sin(\omega t), m = 2, \gamma = 0 \) (no damping), \( k = 8 \), find \( \omega \) which causes the resonance. Find \( u(t) \) for this \( \omega \), if the initial position \( u(0) = 1 \) and the initial speed \( u'(0) = 0 \).

**Problem 7.** Let \( m = 1, k = 4 \). For which \( \gamma \) is there an overdamping? underdamping? Solve for \( \gamma = 1, u(0) = 1, u'(0) = -1 \), and find the amplitude (dependent on \( t \)), and initial phase.

**Problem 8.** Consider the body of mass \( m \) moving in the water. Assume there is no gravity. The water resistance is \( kv^2 \), for \( k > 0 \). The initial speed is \( v_0 \). Find the time when the speed is \( v_0/2 \).

**Problem 9.** Are the following first-order equations linear? separable? autonomous? If they are linear, are they homogeneous or nonhomogeneous? Yes/No.

(i) \( y' = 4y + t \);
(ii) \( ty' = y \);
(iii) \( y' = y^2 \);
(iv) \( yy' = 1 \);
(v) \( y' = y^2 + t \).

**Problem 10.** Using variation of parameters, find the solution to

\[ (1 - t)y'' + ty' - y = 0, \]

given that one solution is \( y_1(t) = t \).

**Problem 11.** A series circuit has a capacitor of 1, a resistor of 3, and an inductor of 2. The initial charge of the capacitor is \(-2\) and there is no initial current. The battery gives \( E(t) = 3 \cos(2t) \). Find the charge \( Q \) on the capacitor at any time \( t \).