Consider the equation
\[ y'' + y' - 2y = e^t. \]

First, solve the corresponding homogeneous equation:
\[ y'' + y' - 2y = 0. \]

Characteristic equation: \( \lambda^2 + \lambda - 2 = 0 \), and the roots are 1 and \(-2\). Therefore, the general solution of the homogeneous equation is
\[ y_0(t) = C_1 e^t + C_2 e^{-2t}. \]

**General Rule.** General solution \( y_0 \) of the homogeneous equation + particular solution \( y_1 \) of the nonhomogeneous equation = general solution of the nonhomogeneous equation.

So to find a general solution of a nonhomogeneous equation, you need to (1) find general solution of the corresponding homogeneous equation, and (2) find any particular solution of the nonhomogeneous equation.

We already did (1). Let us do (2). We could try \( y_1(t) = Ae^t \) for unknown constant \( A \). But this does not work, because this is a part of a general solution to the homogeneous equation: \( C_1 e^t + C_2 e^{-2t} \).

In other words, if you plug in \( y_1 = Ae^t \) into \( y'' + y' - 2y \), you get 0 instead of \( e^t \).

Try \( y_1 = Ate^t \). We get:
\[
\begin{align*}
y_1' &= At'e^t + Ae^t + Ate^t, \\
y_1'' &= A(e^t)' + At'e^t + At(e^t)' = Ae^t + Ate^t + 2Ae^t + Ate^t = 2Ae^t + Ate^t.
\end{align*}
\]

Then
\[ y'' + y' - 2y = 3Ae^t = e^t \Rightarrow 3A = 1 \Rightarrow A = \frac{1}{3}. \]

Therefore,
\[ y_1 = \frac{1}{3} te^t. \]

The general solution of the equation is
\[ \frac{1}{3} te^t + C_1 e^t + C_2 e^{-2t}. \]

If we had
\[ y'' + y' - 2y = (t + 2)e^t, \]
then try
\[ y_1 = (At^2 + Bt)e^t. \]

**General Rule.** If the right-hand side is a polynomial of degree \( n \) times \( e^{at} \), where \( a \) is a root of the characteristic equation, then try \( y_1 = \) polynomial of degree \( n + 1 \) times \( e^{at} \).

If \( a \) is not a root, then do not raise the degree of polynomial.