
Let us show that we can also solve the equations with the continuous right-hand side using the Laplace transform.

Let
\[ y'' + y = e^t, \ y(0) = 1, \ y'(0) = 0. \]

Then
\[ L[y''] + L[y] = L[e^t] = \frac{1}{s - 1}. \]

But
\[ L[y''] = s^2 L[y](s) - sy(0) - y'(0) = s^2 L[y](s) - s. \]

Therefore, we have:
\[ s^2 L[y](s) + L[y](s) - s = \frac{1}{s - 1}. \]

Solving for \( L[y](s) \), we get:
\[ L[y](s) = \frac{s}{s^2 + 1} + \frac{1}{(s - 1)(s^2 + 1)}. \]

The remaining part of the solution will be devoted to the decomposition of the right-hand side into the elementary fractions. First, decompose
\[ \frac{1}{(s - 1)(s^2 + 1)} = \frac{A}{s - 1} + \frac{Bs + C}{s^2 + 1}. \]

Find \( A, B \) and \( C \). Multiply by \( s - 1 \):
\[ \frac{1}{s^2 + 1} = A + \frac{(Bs + C)(s - 1)}{s^2 + 1}. \]

Let \( s = 1 \) and get \( A = \frac{1}{2} \). Therefore,
\[ \frac{1}{(s - 1)(s^2 + 1)} = \frac{1/2}{s - 1} + \frac{Bs + C}{s^2 + 1}. \]

Subtracting, we get:
\[ \frac{1}{(s - 1)(s^2 + 1)} = \frac{1}{2(s - 1)} + \frac{-s - 1}{2(s^2 + 1)}. \]

Therefore,
\[ L[y](s) = \frac{1}{2(s - 1)} + \frac{s - 1}{2(s^2 + 1)}. \]

Recall that
\[ e^t \leftrightarrow \frac{1}{s - 1}, \ \cos t \leftrightarrow \frac{s}{s^2 + 1}, \ \sin t \leftrightarrow \frac{1}{s^2 + 1}. \]

Inverse Laplace transforms:
\[ y(t) = \frac{1}{2} e^t - \frac{1}{2} \sin t + \frac{1}{2} \cos t. \]

The same result can be obtained by using the method of undetermined coefficients. That is, solving the homogeneous equation, and finding a particular solution for the nonhomogeneous equation in the form of \( Ae^t \), then finding the constants using initial conditions.