Problem 1. Solve the following boundary value problem: \( y'' + \lambda y = 0, \; y'(0) = 0, \; y'(L) = 0. \) Write the solution in full detail, discussing all cases.

For Problems 2-6, solve the following initial-boundary value problems for the heat equation:

Problem 2. \( u_t = 4u_{xx}, \; u|_{t=0} = 0, \; u|_{x=0} = u|_{x=2} = 0. \)

Problem 3. \( u_t = u_{xx}, \; u|_{t=0} = 2, \; u_x|_{x=0} = u_x|_{x=1} = 0. \)

Problem 4. \( u_t = \frac{1}{2}u_{xx}, \; u|_{t=0} = \sin^2 x, \; u|_{x=0} = u|_{x=\pi} = 0. \) (Hint: Use Problem 5 from HW 5.)

Problem 5. \( u_t = 3u_{xx}, \; u|_{t=0} = \sin^2 x, \; u_x|_{x=0} = u_x|_{x=\pi} = 0. \) (Hint: Use Problem 6 from HW 5.)

Problem 6. \( u_t = u_{xx}, \; u|_{t=0} = \chi(1,2), \; u_x|_{x=0} = u_x|_{x=3} = 0. \)