Problem 1. Solve the system
\[ x'(t) = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} e^{4t} \\ t e^t \end{bmatrix} \]
(try to solve it both using diagonalization and undetermined coefficients).

Problem 2. Solve the eigenvalue problem for
\[ A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \]

Problem 3. Classify the system
\[ x'(t) = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} x(t), \]
and draw a phase portrait. (Classify means nodal sink/nodal source/saddle point/center/unstable spiral/stable spiral and stable/unstable/asymptotically stable.)

Problem 4. For the matrix
\[ A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \]
find the fundamental matrix \( e^{tA} \) and solve the system \( x'(t) = A x(t) \).

Problem 5. The same problem for
\[ x'(t) = \begin{bmatrix} 1 & -2 \\ 5 & 2 \end{bmatrix} \]

Problem 6. For the function
\[ f(x) = \chi_{[0,1]}(x) = \begin{cases} 1, & 0 \leq x \leq 1, \\ 0, & \text{else}, \end{cases} \]
find: (i) Fourier series on \([0,1]\); (ii) Fourier series on \([0,2]\); (iii) Fourier series on \([0,1/2]\); (iv) Fourier series on \([-2,2]\); (v) sine series on \([0,1]\); (vi) sine series on \([0,2]\); (vii) cosine series on \([0,1]\); (viii) cosine series on \([0,2]\); (ix) to which function does the series from (ii) converge for different \(x\)? (x) the same question for (v).

For Problems 7-12, solve the following initial- and boundary-value problems for the partial differential equations. (For Problems 7-10, use the results of Problem 6.)

Problem 7. \( u_t = 2 u_{xx}, \quad u|_{x=0} = u|_{x=2} = 0, \quad u|_{t=0} = \chi_{[0,1]}(x) \)

Problem 8. \( u_t = 2 u_{xx}, \quad u_x|_{x=0} = u_x|_{x=2} = 0, \quad u|_{t=0} = \chi_{[0,1]}(x) \)

Problem 9. \( u_{tt} = 3 u_{xx}, \quad u_x|_{x=0} = u_x|_{x=2} = 0, \quad u|_{t=0} = \chi_{[0,1]}(x), \quad u_t|_{t=0} = \chi_{[0,1]}(x) \)

Problem 10. \( u_{tt} = 3 u_{xx}, \quad u_x|_{x=0} = u_x|_{x=2} = 0, \quad u|_{t=0} = \chi_{[0,1]}(x), \quad u_t|_{t=0} = \chi_{[0,1]}(x) \)

Problem 11. \( u_{xx} + u_{yy} = 0, \quad u|_{r=2} = \cos(3\theta) + \cos^2(\theta) \)

Problem 12. \( u_{xx} + u_{yy} = 0, \quad u|_{r=1/2} = \chi_{[\pi/2,3\pi/2]}(\theta) \)