Find all eigenvalues and eigenvectors of the matrix $A$. Indicate algebraic and geometric multiplicity of each eigenvalue. Is this matrix defective or not? If it is not defective, diagonalize it.

$$A = \begin{bmatrix} 5 & 1 \\ -1 & 7 \end{bmatrix}$$

**Solution.** Consider the characteristic polynomial

$$\det(A - \lambda I_3) = \begin{vmatrix} 5 - \lambda & 1 \\ -1 & 7 - \lambda \end{vmatrix} = (5 - \lambda)(7 - \lambda) - (-1) \cdot 1 = \lambda^2 - 12\lambda + 36 = (\lambda - 6)^2.$$ 

The eigenvalue is $\lambda = 6$. Eigenvectors: we need to solve the system $Ax = 6x$. We can write it as

$$\begin{cases} 5x_1 + x_2 = 6x_1 \\ -x_1 + 7x_2 = 6x_2 \end{cases} \Rightarrow x_1 = x_2 \quad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The eigenvalue $\lambda = 6$ has only one eigenvector. Its algebraic multiplicity is two, and its geometric multiplicity is one. So the matrix is defective, it is not diagonalizable.