Math 324F Practice Problems for Exams

These are practice problems from Stewart’s book. The numbers of these problems are given according to Calculus, 7th Edition, Early Transcendentals. The number 15.7.13 means Chapter 15, section 7, problem 13. They are all odd-numbered, so that you can look at the solutions manual.

1 Practice Problems for Midterm 1

15.3.19. \( \int \int_D y^2 \text{d}A \), where \( D \) is the triangular region with vertices (0, 1), (1, 2), (4, 1).
15.3.21. \( \int \int_D (2x-y) \text{d}A \), where \( D \) is bounded by the circle with center at the origin and radius 2.
15.3.23. Find the volume of the solid under the plane \( x-2y+z=1 \) and above the region bounded by \( x+y=1 \) and \( x^2+y=1 \).
15.7.13. \( \iiint_E 6xyz \text{d}V \), where \( E \) lies under the plane \( z=1+x+y \) and above the region in the \( xy \)-plane bounded by the curves \( y=\sqrt{x} \), \( y=0 \), \( x=1 \).
15.7.15. \( \iiint_T x^2 \text{d}V \), where \( T \) is the solid tetrahedron with vertices (0, 0, 0), (1, 0, 0), (0, 1, 0) and (0, 0, 1).
15.7.17. \( \iiint_E x \text{d}V \), where \( E \) is bounded by the paraboloid \( x=4y^2+4z^2 \) and the plane \( x=4 \).
15.8.19. \( \iiint_E (x+y+z) \text{d}V \), where \( E \) is the solid in the first octant that lies under the paraboloid \( z=4-x^2-y^2 \).
15.8.21. \( \iiint_E x^2 \text{d}V \), where \( E \) is the solid that lies within the cylinder \( x^2+y^2=1 \), above \( z=0 \) and below \( z^2=4x^2+4y^2 \).
15.8.23. Find the volume of the solid that is enclosed by the cone \( z=\sqrt{x^2+y^2} \) and the sphere \( x^2+y^2+z^2=2 \).
15.9.23. \( \iiint_E (x^2+y^2) \text{d}V \), where \( E \) lies between the spheres \( x^2+y^2+z^2=4 \) and \( x^2+y^2+z^2=9 \).
15.9.25. \( \iiint_E xe^{x^2+y^2+z^2} \text{d}V \), where \( E \) is the portion of the unit ball \( x^2+y^2+z^2\leq1 \) that lies in the first octant.
15.9.27. Find the volume of the part of the ball \( \rho \leq a \) that lies between the cones \( \varphi = \pi/6 \) and \( \varphi = \pi/3 \).
15.10.15. \( \int \int_R (x-3y) \text{d}A \), where \( R \) is the triangular region with vertices (0, 0), (2, 1), and (1, 2); \( x=2u+v, y=u+2v \).
15.10.17. \( \int \int_R x^2 \text{d}A \), where \( R \) is the region bounded by the ellipse \( 9x^2+4y^2=36; x=2u, y=3v \).
15.10.21. \( \int \int_E \text{d}V \), where \( E \) is the solid enclosed by the ellipsoid \( x^2/a^2+y^2/b^2+z^2/c^2=1; x=au, y=bu, z=bu \).
14.5.3. \( z=\sqrt{1+x^2+y^2} \), \( x=\ln t, y=\cos t \). Use the Chain Rule to find \( dz/dt \).
14.5.11. \( z=e^r \cos \theta, r=st, \theta=\sqrt{s^2+t^2} \). Use the Chain Rule to find \( z_u \) and \( z_t \).
14.5.21. \( z=x^4+x^2y, x=s+2t-u, y=stu \). Find \( z_u, z_t, z_u \) when \( s=4, t=2, u=1 \).
14.6.5. Find the directional derivative of \( f \) at the given point to the direction indicated by the angle \( \theta \): \( f(x,y)=ye^{-x}, (0,4), \theta=\pi/3 \).
14.6.15. Find the directional derivative of the function at the given point in the direction of the vector \( \mathbf{v} \): \( f(x,y,z)=xe^y+ye^z+ze^x \), (0, 0, 0), \( \mathbf{v}=<5,1,-2> \).
14.6.21. Find the maximum rate of change of \( f \) at the given point and the direction in which it occurs: \( f(x,y)=4y\sqrt{x} \), (4, 1).
16.2.1. \( \int_C y^3 \text{d}s \), \( C: x=t^3, y=t, 0 \leq t \leq 2 \).
16.2.5. \( \int_C (x^2y^2-\sqrt{x}) \text{d}s \), \( C \) is the arc of the curve \( y=\sqrt{x} \) from (1, 1) to (4, 2).
16.2.11. \( \int_C xe^y \text{d}s \), \( C \) is the line segment from (0, 0, 0) to (1, 2, 3).
2 Practice Problems for Midterm 2

16.6.23. Find a parametric representation for the surface which is the part of the sphere \( x^2 + y^2 + z^2 = 4 \) that lies above the cone \( z = \sqrt{x^2 + y^2} \).

16.6.33. Find an equation of the tangent plane to the given parametric surface at the specified point: \( x = u + v, \ y = 3u^2, \ z = u - v, \ (2, 3, 0) \).

16.6.49. Find the area of the surface \( x = u^2, \ y = uv, \ z = v^2/2, \ 0 \leq u \leq 1, \ 0 \leq v \leq 2 \).

16.7.9. \( \int \int_S x^2yzdS \), where \( S \) is the part of the plane \( z = 1 + 2x + 3y \) that lies above the rectangle \([0, 3] \times [0, 2] \).

16.7.15. \( \int \int_S ydS \), where \( S \) is the part of the paraboloid \( y = x^2 + z^2 \) inside the cylinder \( x^2 + z^2 = 4 \).

16.7.17. \( \int \int_S (x^2z + y^2z)dS \), where \( S \) is the hemisphere \( x^2 + y^2 + z^2 = 4, \ z \geq 0 \).

16.1.3. Sketch the vector field \( \mathbf{F}(x, y) = -1/2i + (y - x)j \).

16.1.5. Sketch the vector field \( \mathbf{F}(x, y) = (yi + xj)/\sqrt{x^2 + y^2} \).

16.1.21. Find the gradient vector field of \( f \): \( f(x, y) = xe^{xy} \).

16.5.5. Find the curl and the divergence of the vector field: \( \mathbf{F}(x, y, z) = xye^zi + yze^zk \).

16.5.17. Determine whether or not the vector field \( \mathbf{F} \) is conservative. If it is conservative, find a function \( f \) such that \( \mathbf{F} = \nabla f \): \( \mathbf{F}(x, y, z) = e^{yz}i + xe^{yz}j + yxe^{yz}k \).

16.2.15. \( \int_C z^2dx + x^2dy + y^2dz \), where \( C \) is the line segment from \((1, 0, 0) \) to \((4, 1, 2) \).

16.2.29. \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y) = e^{x-1}i + xyj \), and \( C \) is given by \( r(t) = t^2i + t^3j, \ 0 \leq t \leq 1 \).

16.2.39. Find the work done by the force field \( \mathbf{F}(x, y) = xi + (y + 2)j \) in moving an object along an arch of the cycloid \( r(t) = (t - \sin t)i + (1 - \cos t)j, \ 0 \leq t \leq 2\pi \).

16.3.13. Find a function \( f \) such that \( \mathbf{F} = \nabla f \), and evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) along the given curve \( C: \mathbf{F}(x, y) = xy^2i + x^2yj, \ r(t) =< t + \sin(\pi t/2), t + \cos(\pi t/2), 0 \leq t \leq 1 \).

16.3.19. Show that the integral \( \int_C 2xe^{-y}dx + (2y - x^2e^{-y})dy \), where \( C \) is any path from \((1, 0) \) to \((2, 1) \), is independent of this path, and find it.

16.3.23. Find the work done by the force field \( \mathbf{F} = 2y^3/2i + 3x\sqrt{y}j \) in moving an object from \( P(1, 1) \) to \( Q(2, 4) \).
3 Practice Problems for the Final Exam

16.7.21. \( \int_{\Sigma} \mathbf{F} \cdot d\mathbf{S}, \mathbf{F}(x, y, z) = ze^{xy}\mathbf{i} - 3ze^{xy}\mathbf{j} + xy\mathbf{k}, \) \( \Sigma \) is the parallelogram with parametric equations \( x = u + v, \ y = u - v, \ z = 1 + 2u + v, \ 0 \leq u \leq 2, \ 0 \leq v \leq 1. \)

16.7.23. \( \int_{\Sigma} \mathbf{F} \cdot d\mathbf{S}, \mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}, \) \( \Sigma \) is the part of the paraboloid \( z = 4 - x^2 - y^2 \) that lies above the square \( 0 \leq x \leq 1, \ 0 \leq y \leq 1 \) and has upward orientation.

16.7.27. \( \int_{\Sigma} \mathbf{F} \cdot d\mathbf{S}, \mathbf{F}(x, y, z) = y\mathbf{j} - zk, \) \( \Sigma \) consists of the part of the paraboloid \( y = x^2 + z^2 \) which satisfies \( 0 \leq y \leq 1, \) and the disc \( x^2 + z^2 \leq 1, \ y = 1. \)

16.4.5. Use Green’s Theorem to evaluate \( \int_{C} xy^2dx + 2x^2ydy, \) \( C \) is the triangle with vertices \((0, 0), \ (2, 2) \) and \((2, 4). \)

16.4.7. The same for \( \int_{C}(y + e^{\sqrt{x}})dx + (2x + \cos y^2)dy, \) \( C \) is the boundary of the region enclosed by the parabolas \( y = x^2 \) and \( x = y^2. \)

16.4.13. The same for \( \int_{C} \mathbf{F} \cdot d\mathbf{r}, \mathbf{F}(x, y) = < y - \cos y, \ x \sin y >, \) \( C \) is the circle \((x-3)^2 + (y+4)^2 = 4 \) oriented clockwise.

16.8.3. Use Stokes’ Theorem to evaluate \( \int_{\Sigma} \text{curl} \mathbf{F} \cdot d\mathbf{S}, \mathbf{F}(x, y, z) = x^2z^2\mathbf{i} + y^2z^2\mathbf{j} + xyz\mathbf{k}, \) \( \Sigma \) is the part of the paraboloid \( z = x^2 + y^2 \) that lies inside the cylinder \( x^2 + y^2 = 4, \) oriented upward.

16.8.9. The same for \( \int_{C} \mathbf{F} \cdot d\mathbf{r}, \mathbf{F}(x, y, z) = yz\mathbf{i} + 2xz\mathbf{j} + e^{xy}\mathbf{k}, \) \( C \) is the circle \( x^2 + y^2 = 16, \ z = 5, \) traversed counterclockwise if viewed from above.

16.8.17. A particle moves along line segments from the origin to the points \((1, 0, 0), \ (1, 2, 1), \ (0, 2, 1) \) and back to the origin under the influence of the force field \( \mathbf{F}(x, y, z) = z^2\mathbf{i} + 2xy\mathbf{j} + 4y^2\mathbf{k}. \) Find the work done.

16.9.5. Use the Divergence Theorem to find \( \int_{\Sigma} \mathbf{F} \cdot d\mathbf{S}, \mathbf{F}(x, y, z) = xye^y\mathbf{i} + xy^2\mathbf{j} - ye^y\mathbf{k}, \) \( \Sigma \) is the surface of the box bounded by the coordinate planes and the planes \( x = 3, \ y = 2, \ z = 1. \)

16.9.7. The same for \( \mathbf{F}(x, y, z) = 3xy^2\mathbf{i} + xe^y\mathbf{j} + z^3\mathbf{k}, \) \( \Sigma \) is the surface of the solid region bounded by \( y^2 + z^2 = 1, \ x = -1, \ y = 2. \)

16.9.13. The same for \( \mathbf{F}(r) = |r|\mathbf{r}, \ r = xi + yj + zk, \) \( \Sigma \) consists of the hemisphere \( z = \sqrt{1 - x^2 - y^2} \) and the disc \( x^2 + y^2 \leq 1 \) on the \( xy \)-plane.