Homework 3, due July 16

Problems from old actuarial exams are marked by a star.

Problem 1*. Upon arrival at a hospital emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

- 10% of the emergency room patients were critical;
- 30% of the emergency room patients were serious;
- the rest of the emergency room patients were stable;
- 40% of the critical patients died;
- 10% of the serious patients died;
- 1% of the stable patients died.

Given that a patient survived, what is the probability that the patient was categorized as serious upon arrival?

Problem 2*. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.

Problem 3*. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Calculate the number of blue balls in the second urn.

Problem 4*. The probability that a visit to a primary care physicians (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCPs office, 30% are referred to specialists and 40% require lab work. Determine the probability that a visit to a PCPs office results in both lab work and referral to a specialist.

Problem 5*. An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the companys policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policyholder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year. A policyholder dies in the next year. What is the probability that the deceased policyholder was ultra-preferred?

Problem 6*. For two events $A$ and $B$, you are given

$$P(A \cup B) = 0.7, \quad P(A \cup B^c) = 0.9.$$ 

Determine $P(A)$. 

Problem 7*. An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is 1/3. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?

Problem 8*. A large pool of adults earning their first drivers license includes 50% low-risk drivers, 30% moderate-risk drivers, and 20% high-risk drivers. Because these drivers have no prior driving record, an insurance company considers each driver to be randomly selected from the pool. This month, the insurance company writes 4 new policies for adults earning their first drivers license. What is the probability that these 4 will contain at least two more high-risk drivers than low-risk drivers?

Problem 9*. An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are 1/4, 1/3, and 5/12, respectively. Determine the probability that a randomly chosen employee will choose no supplementary coverage.

Problem 10*. A random person has a heart disease with probability 25%. A person with a heart disease is a smoker with probability twice as a person without a heart disease. What is the conditional probability that a smoker has a heart disease?

Problem 11. Toss three fair coins. Are the following events A and B independent?
   (i) A = \{first and second tosses resulted in exactly one H\}, B = \{second and third tosses resulted in exactly one H\};
   (ii) A = \{first and second tosses resulted in exactly one H\}, B = \{second and third tosses resulted in at least one H\};
   (iii) A = \{first and second tosses resulted in at least one H\}, B = \{second and third tosses resulted in at least one H\};
   (iv) A = \{first toss in H\}, B = \{second and third tosses resulted in at least one H\}.

Problem 12. Genes related to albinism are denoted by A (non-albino) and a (albino). Each person has two genes. People with AA, Aa or aA are non-albino, while people with aa are albino. The non-albino gene A is called dominant. Any parent passes one of two genes (selected randomly with equal probability) to his offspring. If a person has either Aa or aA, then he is called a carrier: he is a non-albino, but he can carry the albino gene to his offspring.
   (i) If a person is non-albino, what is the conditional probability that he is a carrier?
   (ii) If a couple consists of a non-albino and an albino, what is the probability that their offspring will be albino?
   (iii) Suppose that a couple consists of two non-albinos. They have two offspring. What is the probability that both offspring are albinos?
**Problem 13.** Is it right that the following statement are true for all possible events $A$, $B$, $C$? (Venn diagrams might be helpful.)

(i) $A \subseteq A \cup B$;
(ii) $A \subseteq A \cap B$;
(iii) $A^c \cap A = \emptyset$;
(iv) $A^c \cup A = \Omega$;
(v) $A^c = \Omega \setminus A$;
(vi) $(A \setminus B) \cup (B \setminus A) = A \cup B$;
(vii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$;
(viii) $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$.

**Problem 14.** Consider two coins: a fair one and a magic one, which always falls H. Suppose you pick a coin at random (each coin with equal probability $1/2$) and toss it $n$ times. All of the tosses are heads. You would like to convince me that the coin is magic, but I will believe you only if the probability that it is magic (given the result that all $n$ tosses are H) exceeds 99%. How large the number $n$ of tosses should be?

**Problem 15.** Let $\Omega = \{1, 2, \ldots, 8\}$, $A = \{2, 4, 6, 8\}$, $B = \{1, 2, 3, 4\}$, $C = \{1, 3, 5, 7\}$. Find the following events:

(i) $A \cup B$;
(ii) $(A \cup B \cup C)^c$;
(iii) $(A \setminus B) \cap C$;
(iv) $A \setminus C$;
(v) $C \setminus A^c$. 