Problem 1. (i) Find $P(X \leq 2)$ for $X \sim \text{Geo}(1/2)$.
(ii) Find $P(X \leq 5)$ for $X \sim \text{NB}(3, 1/3)$.
(iii) Find $P(X \geq 3)$ for $X \sim \text{NB}(3, 1/3)$.
(iv) Find $P(X = 0)$ for $X \sim \text{Geo}(1/4)$.

Problem 2. Let $X, Y, Z$ be random variables such that

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>Z</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/8</td>
</tr>
<tr>
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<td>0</td>
<td>1</td>
<td>1/8</td>
</tr>
<tr>
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<td>1</td>
<td>2</td>
<td>1/8</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>1/8</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
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(i) Find the distribution and the expectation of each of these three random variables.
(ii) Find Cov($X, Y$), Cov($X, Z$) and Cov($Y, Z$).
(iii) Find the distribution of $XY + Z$.
(iv) Calculate $E(X + 1)^4$.
(v) Calculate $E(XYZ)$.

Problem 3. While answering a question on a multiple-choice test with 5 options for this question, a student knows the answer with probability 70%. If he does not know the answer, he randomly chooses the answer (with probability 1/5 for each answer).
(i) Assume that the answer is correct. What is the probability that he knew the answer?
(ii) Assume that there are 10 questions, and the student did all of them correctly. What is the probability that he knew 9 or more of them?

Problem 4. An insurance company sells car insurance, with an obligation to pay 100,000$ in case of an accident. There are four categories of people:

- teenagers: $N_1 = 1500$, $p = 0.05\%$;
- young adults with college education: $N_2 = 2000$, $p = 0.01\%$;
- young adults without college education: $N_3 = 1500$, $p = 0.03\%$;
- middle-aged people: $N_4 = 3000$, $p = 0.02\%$.

The first number refers to the quantity of people in each category. The second number refers to the probability of an accident for a given person in this category. The company wants to pay all the claims using money collected from premiums. It wants to be able to do this with probability 90% or more.

The CEO of the company decides that a fair rule to collect premiums is as follows: people in each category should pay proportionally to the probability $p$ of an accident. For example, teenagers should pay five times as more as young adults with college education. What exactly should the premiums be for each group?
**Problem 5**. A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 3/5. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months. Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs.

**Problem 6**. An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. What is the expected benefit under this policy?

**Problem 7**. For two independent random variables $X$ and $Y$, we have
\[ \text{Var}(X + Y) = \text{Var} X + \text{Var} Y. \]
Why is this?

**Problem 8**. How do we define $X \sim \text{Bin}(N, p)$? Let $q = 1 - p$. Show why the following formulas are true:
\[
P(X = k) = \binom{N}{k} p^k q^{N-k}, \quad k = 0, \ldots, N,
\]
\[ \mathbb{E}X = Np, \quad \text{Var} X = Npq. \]

**Problem 9**. (St. Petersburg Paradox.) You are invited to play a game. You toss a coin until you get the first Heads. If the first Heads is at the $n$th toss, then you win $2^n$. What is the expected value of the amount you win?

Problems from old actuarial exams are marked by a star.