Problem 1. You have three coins, two of them fair and the third a magic one, which rolls out Heads with probability 75%. Suppose you picked randomly one coin and tossed it once. You got Heads. What is the probability that this is the magic coin?

Solution. Let $F_1 = \{\text{coin is magic}\}$, $F_2 = \{\text{coin is fair}\}$, $A = \{\text{Heads}\}$. Then

$$
P(F_1) = \frac{1}{3}, \quad P(F_2) = \frac{2}{3}, \quad P(A | F_1) = \frac{3}{4}, \quad P(A | F_2) = \frac{1}{2}.
$$

By the second Bayes’ formula, we have:

$$
P(F_1 | A) = \frac{P(F_1)P(A | F_1)}{P(F_1)P(A | F_1) + P(F_2)P(A | F_2)} = \frac{(1/3)(3/4)}{(1/3)(3/4) + (2/3)(1/2)} = \frac{3}{7}.
$$

Problem 2. Old license plates consist of six symbols: three digits and three numbers (at any place, not necessarily digits first). Symbols cannot repeat. How many such plates? Do not just write numbers; give explanations.

Solution. We can find three slots for digits in $\binom{6}{3} = (6 \cdot 5 \cdot 4)/6 = 20$ ways. The first digit can be chosen in 10 ways, the second digit in 9 ways, the third one in 8 ways. The first letter can be chosen in 26 ways, the second letter in 25 ways, the third one in 24 ways. The answer is

$$20 \cdot 10 \cdot 9 \cdot 8 \cdot 26 \cdot 25 \cdot 24$$