Problem 1. You take five preliminary exams, and you will pass each exam with probability \( \frac{1}{3} \), independently of other exams. What is the probability that you will pass two or more exams?

Solution. The number \( X \) of passed exams has the Binomial distribution \( X \sim \text{Bin}(5, \frac{1}{3}) \). Therefore,

\[
P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - \left(1 - \frac{1}{3}\right)^5 - \binom{5}{1} \left(1 - \frac{1}{3}\right)^4 \frac{1}{3}
\]

\[
= 1 - \left(\frac{2}{3}\right)^5 - 5 \left(\frac{2}{3}\right)^4 \frac{1}{3} = 0.54
\]

Problem 2. Toss a fair coin three times. Let \( X \) be the number of Heads during the first two tosses. Let \( Y \) be the number of Tails during the last two tosses. For example, if the sequence of tosses is TTH, then \( X = 0 \) and \( Y = 1 \). Are \( X \) and \( Y \) independent? Why or why not?

Solution. They are dependent, because \( P(X = 0) = \frac{1}{4} \) and \( P(Y = 0) = \frac{1}{4} \), but \( P(X = 0, Y = 0) = 0 \). Indeed, the second toss must be either Heads or Tails, which will count in either \( X \) or \( Y \): either \( X \geq 1 \) or \( Y \geq 1 \) (or both).