1. Suppose that $L(X_n) \Rightarrow \delta_c$ for some $c \in \mathbb{R}$. Prove that $\{X_n\}$ converges to $c$ in probability. In addition, explain the general relationship between weak convergence and convergence in probability.

2. Prove that any finite collection of probability measures $\{\mu_n\}_{n=1}^N$ is tight.

3. Suppose that for $n \in \mathbb{N}$, we have $P(X_n = 5) = 1/n$ and $P(X_n = 6) = 1 - 1/n$.
   (a) Compute the characteristic function $\phi_{X_n}(t)$ for all $n \in \mathbb{N}$ and $t \in \mathbb{R}$.
   (b) Compute $\lim_{n \to \infty} \phi_{X_n}(t)$.
   (c) Specify a distribution $\mu$ such that $\lim_{n \to \infty} \phi_{X_n}(t) = \int e^{itx} d\mu(x)$ for all $t \in \mathbb{R}$.
   (d) Determine whether or not $L(X) \Rightarrow \mu$.

4. Let $\{X_n\}$ be a sequence of i.i.d. random variables with $E[X_i] = 3$ and $\text{Var}(X_i) = 4$ for all $i \in \mathbb{N}$. Let $S = X_1 + X_2 + \cdots + X_{10,000}$. In terms of $\Phi(x)$, the CDF for the standard normal distribution, give an approximate value for $P(S \leq 30,500)$. Justify your answer.

5. Give an example of the following (specify all necessary items given in the corresponding definitions and justify your examples):
   (a) A Markov chain.
   (b) A stochastic process that doesn’t satisfy the Markov property.
   (c) A martingale that is also Markovian.
   (d) A martingale that is non-Markovian.

6. Let $Z \geq 0$ be a random variable defined on a probability space $(\Omega, \mathcal{F}, P)$ with $E[Z] = 1$. Define a function $P' : \mathcal{F} \to \mathbb{R}$ by
   $$P'(A) = E[1_A Z] = \int_A Z dP$$
   for all $A \in \mathcal{F}$. Show that $P'$ is a probability measure.