Due on Thursday November 9 at the beginning of lecture. Problems are from the Probability Models book (same numbering in both the 10th and 11th editions).

1. Chapter 5, Problem 56
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3. Simulate a nonhomogeneous Poisson process with intensity function
   \[ \lambda(t) = 1 + 5(1 + \sin(\pi t)). \]
   (a) Generate a sample path and include a figure for the interval \([0,10]\).
   (b) Compute the average value of \(\lambda(t)\) over \([0,10]\) and simulate a **homogeneous** Poisson process with this average rate.
   (c) Generate a sample path of the HPP to compare with the NHPP version above (include a figure).
   (d) Briefly discuss your observations.
4. Let \(N(t)\) be a Poisson process with rate \(\lambda\), and let \(Y_1, Y_2, \ldots\) be \(i.i.d\). random variables with mean \(\mu\) and variance \(\sigma^2\). Derive the mean and the variance of the compound Poisson random variable
   \[ Z(t) = \sum_{i=1}^{N(t)} Y_i. \]
5. Give an example of a compound Poisson process \(\{Z(t) : t \geq 0\}\) where \(Z(t)\) is defined as in Problem 3. Make sure to explain what \(N(t), Y_i\) and \(Z(t)\) stand for in the example.
6. Write an algorithm in R to simulate a linear birth and death process such that the total birth rate is \(\lambda_i = i\lambda\) and the total death rate is \(\mu_i = i\mu\). Use \(\lambda = 1, \mu = 0.5\), and start with 100 individuals in the population.
   (a) Generate three sample paths of the process and include a figure (for time interval \([0,100]\)).
   (b) What happens if the death rate is increased to \(\mu = 2\)? Include another figure with three sample paths for this case.
   (c) Briefly describe your observations of the processes in (a) and (b).
   (d) **BONUS:** Modify the algorithm above to include immigration at an exponential rate \(\theta\). Generate sample paths for 3 different values of \(\theta\), include R code, parameter values, and a figure.

Include your R code along with the answers to the items listed above.