Frequency Response
Controller Design

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Outline

• Advantages/disadvantages.
• Design procedures.
• Examples.
Frequency Response Design

**Advantage:** Familiar design procedure.

**Disadvantages:**

1. Indirect design: controller distortion.
2. Requires experience.
3. Familiar criteria (PM, GM) have different values from their analog counterparts for the same performance.
Design Procedure

1. Select $T$ and obtain the transfer function $G_{ZAS}(z)$.

2. Bilinearly transform $G_{ZAS}(z)$ into $G(w)$ using

$$z = \frac{1 + \frac{wT}{2}}{1 - \frac{wT}{2}}$$

MATLAB

```matlab
>> gd = c2d(g, T)
>> gw = d2c(gd, 'tustin')
```
3. Draw the Bode plot of $G(j\nu)$, and use analog frequency response methods to design a controller $C(w)$ that satisfies the frequency domain specifications.

4. Transform the controller back into the $z$-plane using $C(z) = C(w)\left|_{w=\frac{2[z-1]}{T[z+1]}}\right.$

5. Verify that the performance obtained is satisfactory.
Design Specifications

• PM, GM, BW
• Use frequency response design procedures for analog systems (e.g. lag, lead, lag-lead).
• $PM \approx 100\zeta$
• $\omega_{gc}$ = gain crossover (0 dB magnitude) frequency
• BW increases with $\omega_{gc}$
Example 6.15

Consider the cruise control system of Example 3.2, where the analog process is

$$G(s) = \frac{1}{s + 1}$$

Transform the corresponding $G_{ZAS}(z)$ to the $\omega$-plane by considering both $T = 0.1$ and $T = 0.01$. Evaluate the role of the sampling period by analyzing the corresponding Bode plots.
Solution

\[ G_{ZAS}(z) = (1 - z^{-1}) \mathcal{Z}\left\{ \frac{G(s)}{s} \right\} \]

MATLAB: \[ >> \text{gd} = \text{c2d}(g, T) \]

- \( T = 0.1 \, \text{s} \), obtain z-transfer function
  \[ G_{ZAS}(z) = \frac{9.516 \times 10^{-2}}{z - 0.9068} \]

- \( T = 0.01 \, \text{s} \), obtain z-transfer function
  \[ G_{ZAS}(z) = \frac{9.95 \times 10^{-3}}{z - 0.99} \]
Bilinear Transformation

\[ \text{>> d2c(gd,'tustin')} \]

- \( T = 0.1 \text{ s} \)
  \[ G_{ZAS}(z) = \frac{9.516 \times 10^{-2}}{z - 0.9068}, G_1(w) = \frac{1 - 0.05w}{w + 1} \]
- \( T = 0.01 \text{ s} \)
  \[ G_{ZAS}(z) = \frac{9.95 \times 10^{-3}}{z - 0.99}, G_2(w) = \frac{1 - 0.005w}{w + 1} \]

\( G_1(w) \) & \( G_2(w) \) have

1. pole at \(-1\) like the \( G(s) \) pole in the \( s\)-plane
2. TF zero while \( G(s) \) does not
Bode Plots $G(s)$, $G_1(w)$ & $G_2(w)$
Discussion

For $G_1(w) & G_2(w)$

- Different frequency response from the analog system
- Greater influence of the zero on the frequency response (system dynamics) when the sampling period is larger.
- Distortion in the low frequency range is negligible.
- Gain as $w$ goes to zero is unity as is the DC gain of the analog system.
Example 6.16

DC motor speed control system: (type 0) analog plant has the transfer function

\[ G(s) = \frac{1}{(s + 1)(s + 10)} \]

Design a digital controller by using frequency response methods to obtain: (i) zero steady-state error due to a unit step, (ii) an overshoot less than 10\%, (iii) a settling time of about 1
Solution

• For 10% overshoot, $T_s = 1 \text{ s}$ we calculate

$$
\zeta = \frac{|\ln(0.1)|}{\sqrt{|\ln(0.1)|^2 + \pi^2}} \approx 0.6
$$

$$
\omega_n = \frac{4}{\zeta T_s} \approx 6.7 \text{ rad/s}
$$

$$
\omega_n > \omega_d \& \frac{2\pi}{6.7 \times 40} = 0.0234
$$

• Choose $T = 0.02 \text{ s}$
Transformation

• MATLAB: \[ \gg \text{gd}=\text{c2d}(g,T) \]

\[
G_{ZA5}(z) = (1 - z^{-1})Z \left\{ \frac{G(s)}{s} \right\} = 1.8604 \times 10^{-4} \frac{z+0.9293}{(z-0.8187)(z-0.9802)}
\]

\[ \gg \text{d2c}(gd,'\text{tustin}') \]

\[
G(w) = \frac{-3.6519 \times 10^{-6}(w + 2729)(w - 100)}{(w + 9.967)(w + 1)}
\]
Bilinear Transformation

\[ G(w) = \frac{-3.6519 \times 10^{-6} (w + 2729)(w - 100)}{(w + 9.967)(w + 1)} \]

- \( PM = 60^\circ \), pole-zero cancellation (simple)

\[ C(w) = 54 \frac{w + 1}{w}, \quad \frac{2}{T} = \frac{2}{0.02} = 100 \]

\[ >> \text{Cd} = \text{c2d(C,.02,'tustin')} \]

\[ C(z) = C(w) \bigg|_{w=\frac{2}{T} \left[ \frac{z-1}{z+1} \right]} = 54 \frac{1.01z - 99}{z - 1} \]

\[ = \frac{54.54z - 53.46}{z - 1} \]
Bode Diagram

\[ G_m = 25.7 \text{ dB (at 32.2 rad/sec), } P_m = 61.3 \text{ deg (at 4.86 rad/sec)} \]

\[ C(w)G(w) \]

\[ G(w) \]
Example 6.17

DC motor speed control system: (type 0) analog plant has the transfer function

\[ G(s) = \frac{1}{(s + 1)(s + 3)} \]

Design a digital controller by using frequency response methods to obtain: (i) zero steady-state error due to a unit step, (ii) an overshoot less than 10%, (iii) a settling time of about 1

Use \( T = 0.2 \text{s} \)
Solution

• TF of analog system, ADC and DAC

\[ G_{ZAS}(z) = (1 - z^{-1})z \left\{ \frac{G(s)}{s} \right\} \]

\[ = 1.5437 \times 10^{-2} \quad \frac{z+0.7661}{(z-0.8187)(z-0.5488)} \quad \frac{(w + 75.5)(w - 10)}{(w + 2.913)(w + 0.9967)} \]

• Poles almost in the same locations as poles of \( G(s) \).

• Consider RHP zero at \( w = 10 \) to make the gain crossover frequency about 5 rad/s.
Design

• Cancel two poles with zeros.
• Add poles at zero and \(-20\)

\[
C(w) = 78 \frac{(w + 2.913)(w + 0.9967)}{w(w + 20)}
\]

\[
C(z) = C(w) \bigg|_{w=\frac{2}{T}\left[\frac{z-1}{z+1}\right]}
\]

\[
= \frac{36.9201z^2 - 50.4902z + 16.5897}{(z - 1)(z - 0.3333)}
\]
Bode plots of $C(w)G(w)$ & $G(w)$

Bode Diagram

$G_m = 9.7$ dB (at 18.2 rad/sec), $P_m = 59.9$ deg (at 3.99 rad/sec)
Closed-loop Step Response

Step Response

System: Gcl
Peak amplitude: 1.08
Overshoot (%): 8.09
At time (sec): 0.4
System: Gcl
Settling Time (sec): 0.718